

# RADAR SURVEYS OF THE SOLAR SYSTEM

JOHN V. EVANS

Staff Member, Lincoln Laboratory, Massachusetts Institute of Technology

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## I. INTRODUCTION

It is probably true to say that more has been learned about the moon and planets in the last decade than in the century that preceded it. This rapid increase in our knowledge has arisen largely as the result of the introduction, nearly simultaneously, of two new and powerful methods of study: deep-space probes and ground-based radar observations. Though the possibility of obtaining radio reflections from the moon and planets was discussed some forty years ago, the equipment needed for such experiments was not constructed until instruments could be sent into the vicinity of these objects by means of large rockets. The first deep-space probes were in fact launched by rockets designed to carry warheads, and it was as a countermeasure to such use that radars with adequate capability to detect planetary reflections were first constructed. Later, the need to communicate with these probes over vast distances in space stimulated the design of other high-power radio systems that were also capable of studying the planets by radio reflections.

This paper provides a brief account of one aspect of the radar investigations, namely, the study of planetary motions. After several centuries of optical observations of the celestial motions of the planets, one is compelled to ask what, if anything, can radar add that is new? Although optical astronomy has developed precise methods of determining angular distances between objects, it has never achieved comparable accuracy in distance measurement. This is a direct consequence of the short baseline available to terrestrial observers. Thus, optical observations have permitted good determinations of the elements of the orbits of the planets, but the sizes of these orbits in terms of terrestrial units have remained poorly known. Radar, on the other hand, has rather poor angular resolution, but unparalleled accuracy in determining distances and velocities. Thus, as we shall show, the two methods (old and new) complement one another well.

In earlier centuries, celestial mechanics profited considerably by the practical need for accurate time keeping and navigation. However, by the beginning of the twentieth century this science had fallen into decline and astronomers by and large were engaged in other more challenging work. Now, with the advent of satellites, deep-space probes and planetary radar there has been a revival of interest. Yet, granted that radar can contribute, one may still ask—why bother? There are two reasons why it seems worth while to develop a better knowledge of planetary motions. This first is simply the practical one that in order to direct a space-probe to the surface, say of Venus, it is necessary to know the precise distance in *terrestrial* units, since the thrust of the rocket motors and the velocity of the vehicle are known only in these units. The second reason is that by making observations of ever greater precision, man can test the validity of the underlying physical laws.

In order to appreciate the contribution played by radar, it is well to understand the limitations of the technique. Thus, in the section that follows, we discuss radar detectability of the planets, and the factors that tend to govern the extent to which we can measure their distances. Section III gives a brief history of the radar detection of Venus, Mars, Mercury, and Icarus and discusses the techniques employed for accurate range and velocity measurements. Section IV discusses theoretical predictions of the range and velocity with which the observations must be compared. The predictions of theory must be worked out to better accuracy than the measurements if the underlying physical laws are to be tested. Thus, each advance in the radar measurements has of necessity been paralleled by a corresponding advance in the accuracy to which the consequences of the theory must be computed. In the space of only a few years of radar observations, it has been possible to show that predictions based upon Einstein's general theory of relativity are in better accord with the observations than predictions that depend

upon Newton's laws of gravitation and the assumption that the velocity of light is constant (Shapiro *et al.*, 1968). This and other conclusions based upon the observations accrued to date are presented in Section V.

## II. THE RADAR DETECTABILITY OF PLANETS

### A. THE RADAR EQUATION

As will be shown below, radar investigation of celestial objects seems forever confined to the solar system. This is to be contrasted with radio astronomy, where objects on the very fringe of the visible universe may be detected. In part, this difference arises because in a radar experiment the echo power decreases as the fourth power of the distance of the target, whereas for straight reception the received power varies inversely as the square of the distance. Also important is the fact that man is not capable of building transmitters as powerful as some of the naturally occurring cosmic transmitters.

The echo power  $P_r$  received in a radar experiment is specified by the radar equation and is usually written

$$P_r = \frac{P_t G A \sigma}{(4\pi R^2)^2} \quad (1)$$

where  $P_t$  is the transmitter power,  $G$  the gain of the transmitting antenna (over isotropic) in the direction of the target,  $A$  the collecting area of the receiving antenna,  $R$  is the range to the target, and  $\sigma$  is termed the target cross section (below). Typical values for these terms might be  $P_t = 100$  kw.,  $G = 10^5$ , and  $R \sim 150 \times 10^6$  km. (one astronomical unit). At this distance (equal to that of the sun) a planet the size of Venus intercepts a total of about 1 watt of power. Approximately one tenth of the incident power is reflected so that back at earth there will be a flux of  $\sim 4 \times 10^{-24}$  watt/meter<sup>2</sup>. Taking a value for  $A = 10^8$  meter<sup>2</sup>, we find  $P_r \sim 4 \times 10^{-21}$  watt.

This received power can be detected only after it has been amplified to work a recording device. In order to be recognized, it must compete successfully with the random radio noise energy collected by the antenna from the sky background and generated internally within the receiver itself. We can add these noise components together to yield a total *equivalent* competing noise power  $P_n$  available at the receiver input terminals where

$$P_n = k T_s b \text{ watts} \quad (2)$$

in which  $k$  is Boltzmann's constant ( $= 1.38 \times 10^{-23}$  joule/°K),  $T_s$  is the system temperature, defined by Eq. (2), and  $b$  is the over-all bandwidth (Hz.) of the receiver.

By employing this approach, we have for convenience assumed that the receiver is noise-free and that all the noise appearing at the output terminals is due to an amount  $P_n$  present at the input terminals. This fiction is useful as it enables us to introduce a ratio  $P_r/P_n$  called the signal-to-noise ratio that exists at the input terminals and remains unaltered by subsequent amplification. To illustrate the value of  $P_r/P_n$  that might be encountered, we shall suppose that in Eq. (2)  $T_s = 100^\circ$  K,  $b = 100$  Hz., so that  $P_n = 1.4 \times 10^{-19}$  watt. We see that for the case we have considered  $P_r/P_n \sim 3 \times 10^{-2}$ . At first sight, it would appear that the echo power cannot be detected in this hypothetical case, since it is less than the noise power. By continuing the observations for a long period of time, however, the echo power can be recognized, provided that the signal-to-noise ratio is not substantially smaller than the value given above.

The detection of signals weaker than noise is a routine operation in almost all radar astronomy experiments. It is referred to as integration and may be carried out all day, or even over several days if need be. Such long integration times require the construction of stable, drift-free processes. Usually these are implemented by converting the signals to digital numbers, which are then handled in an electronic computer. It also means that one must know *a priori* the frequency to which the receiver must be tuned to obtain an echo. This will not be the same as the transmitted frequency, because of Doppler effects. Where pulses are used one must, in addition, compensate for the slow change in the flight time of the signals so that in the integration process successive pulses are added one on top of another and not smeared out.

### B. THE TARGET CROSS SECTION

The term  $\sigma$  depends on both the size and the scattering properties of the target. In the case of perfectly reflecting sphere of radius  $r$  (where  $r$  is much greater than the radio wave length  $\lambda$ ), the cross section would be  $\sigma = \pi r^2$ . The power intercepted by such a sphere would be reradiated isotropically. It follows that we may define  $\sigma$  as the projected area presented by a perfectly reflecting sphere which, when placed in the same position as the actual target, would scatter as much power

back to the observer. If the sphere were replaced by one made of dielectric materials, then the reflected power would no longer be reradiated isotropically and the cross section becomes  $\sigma = \rho_0 \pi r^2$ , where  $\rho_0$  is the Fresnel reflection coefficient at normal incidence. For a lossless dielectric material,  $\rho_0 = (1 - \sqrt{\epsilon})^2 / (1 + \sqrt{\epsilon})^2$ , where  $\epsilon$  is the dielectric constant of the material. If the surface is not lossless, then the reflection coefficient depends also upon the electrical conductivity and will decrease with increasing frequency. If the sphere is irregular, the echo power will be a function of the aspect from which it is viewed. In some directions, the reflections from certain portions will add coherently, but when viewed from other aspects, these elements will contribute signals that destructively interfere. Thus, for targets like the moon and the terrestrial planets the echoes are subject to deep fading, and when we discuss the target cross section we mean the average value, i.e., that obtained by determining the echo power averaged over a long time interval. Since it is usually necessary to integrate many echoes in order to secure detection, the short-period fading of planetary echoes has not often been observed directly.

In the case of an irregular dielectric sphere, the cross section  $\sigma$  can be written  $\sigma = g \rho_0 \pi r^2$ , where  $g$  is the factor that allows for any preferential back-scattering introduced by the surface structure. For a sphere that is smooth and undulating with a surface that has a root mean square slope  $\alpha$ , the value of  $g$  is  $1 + \alpha^2$ . For a surface that is rough on the scale of a wave length, it is expected that  $g > 1$ , but precise estimates become difficult to obtain, as the theory of the scattering mechanism has not been worked out for such cases. Usually the product  $g \rho_0$  is lumped together and is reported in the literature as a percentage cross section. The values depend somewhat on wave length, but useful averages are 7 per cent for the moon, 15 per cent for Venus, 6 per cent for Mercury and 8 per cent for Mars (Pettengill, 1968).

### C. THE PATH LOSS

The radar equation [Eq. (1)] can be written in a way that groups together separately those parameters that are within the designer's control and those that are not; thus

$$P_r = [P_t G A] \left[ \frac{\sigma}{(4\pi R^2)^2} \right] \text{watts} \quad (3)$$

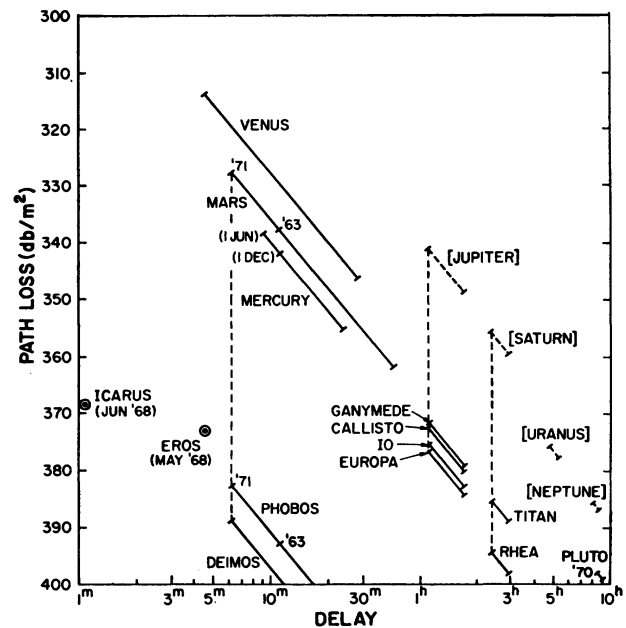


FIG. 1. The estimated radar detectability of a number of possible targets in the solar system shown in terms of their logarithmic path loss and plotted against the round-trip radar echo delay. The variation in path loss and delay for a given target reflects its varying distance from earth. Targets shown with dotted lines have deep atmospheres and consequently have radar scattering properties which are difficult to predict. They are shown for reference under the (unlikely) assumption that they have reflectivity as high as 0.1.

The term in the second pair of brackets specifies what fraction of 1 watt radiated by an isotropic antenna would be available after reflection by the target to a receiving antenna with a collecting area of 1 m.<sup>2</sup> This term is, therefore, called the *path loss*, and when specified in this way is not directly a function of the operating wave length. In the case of the moon at its average distance, the path loss is  $1.9 \times 10^{-25}$  m.<sup>-2</sup> (or  $-247$  db. per m.<sup>2</sup>). The moon's motion introduces changes in this value of only  $\pm 1$  db. The next lowest value of path loss is that encountered for Venus at inferior conjunction, namely,  $-314$  db. per m.<sup>2</sup> Figure 1 shows the path loss encountered for Venus, Mercury, and Mars as a function of the time of flight for a pulse to the planet and back.

Realistic values for the path loss to the major planets Jupiter, Saturn, Uranus, and Neptune are hard to arrive at. Those presented in figure 1 assume a reflection coefficient  $\rho_0 = 0.1$ . However, these planets are surrounded by dense atmospheres which probably cause considerable attenuation of radio signals propagating inward toward a liquid

TABLE 1  
COMPARISON OF SOME RADAR ASTRONOMY SYSTEMS

Organization	Location	Fre- quency (Mc./s.)	Antenna			Mean power (kw.)	Peak power (kw.)	Pulse length	System tempera- ture (°K)
			Diam. (ft.)	Gain (db.)	Aper- ture (m. <sup>2</sup> )				
Calif. Inst. Technol., JPL. Cornell Univ.	Goldstone Lake, Calif. Arecibo, Puerto Rico	2,388	85	54.2	355	400	400	CW <sup>a</sup>	30
		430	1,000	56.0	16,000	100	2,000	0.03–10.0 msec.	200
Mass. Inst. Technol. Lincoln Lab.	Westford, Mass. (Millstone)	1,295	84	47.3	220	150	5,000	0.04–4.0 msec.	80
Mass. Inst. Technol. Lincoln Lab.	Tyngsboro, Mass. (Haystack)	7,750	120	66.1	482	300	300	CW	60

<sup>a</sup> CW means continuous-wave.

or solid surface. Radar detection of Jupiter has been claimed by two groups (Goldstein, 1964a; Kotel'nikov *et al.*, 1964) but repeatable results have not yet been obtained, so that it is difficult to judge the reliability of these claims or decide upon the possible reflection mechanism. Other workers have searched for echoes from Jupiter and concluded that the value of  $\rho_0 g$  cannot exceed 0.043 per cent at a wave length of 70 cm. (Dyce *et al.*, 1967).

Some of the minor planets should also be detectable when they are near the earth, and figure 1 shows the path loss to Icarus and Eros for the times of their close encounters in 1968. It is a measure of the performance of existing radar systems that Icarus was detected and Eros was considered below the present level of detectability so that no attempt was made to observe it (Section III).

It must be noted that figure 1 is based upon the assumption that the planet has a small or zero angular rotation with respect to the observer. If this is not the case, the effect of the Doppler dispersion of the signals will be to lower the detectability.

#### D. RADAR SYSTEMS

The strength of the echo is determined by the product  $P_t G A$  in Eq. (1), whilst the strength of the competing noise is determined by  $T_s b$  in Eq. (2). These five parameters are (to a limited extent) within the designer's control and determine how well the radar performs. The term  $A$  is not the physical aperture of the antenna, because this would require an efficiency of 100 per cent. Most antennas have a collecting efficiency in the range 50 to 70 per cent, and  $A$  is the effective collecting

area. For parabolic reflecting antennas, which are the type most widely used,  $G$  and  $A$  are related through  $G \approx 4\pi A/\lambda^2$ , where  $\lambda$  is the radio wave length. Therefore, if the same antenna is used for both transmitting and receiving, the signal-to-noise ratio depends upon  $P_t A^2/\lambda^2 b T_s$ . Thus the most important parameter is the antenna size, since the echo intensity increases as the fourth power of the reflector radius. Most of the increase in performance obtained in recent years has been achieved by using extremely large antennas. For example, the Arecibo Ionospheric Observatory (table 1) employs an antenna with a diameter of 1,000 feet. Of next greatest importance is the operating wave length,  $\lambda$ . The antenna gain will continue to rise as  $\lambda$  is reduced until the surface irregularities approach  $\sim \lambda/12$  in size. The rays entering the mirror are then no longer brought to a single focus. Any further reduction in wave length will cause the gain to be reduced. Thus, we require that the antenna both be large and have an accurate surface. These, of course, tend to be mutually opposed requirements.

Table 1 lists the parameters of four of the most powerful radar astronomy stations presently in existence. The CW transmitters listed yield a peak power that is no greater than the average. Pulse transmitters, on the other hand, are usually capable of radiating for short intervals at a level of some 10 or 20 times the mean power. This capability is advantageous when observing targets like Mars where the rotation rate is high and therefore there is considerable Doppler dispersion (Pettengill, 1968).

The installation operated by the Jet Propulsion Laboratory (JPL) has for some time employed a single 85-foot-diameter telescope with a Cassegrain

feed arrangement. More recently, a larger antenna 210 feet in diameter has been employed for reception, and when the functions of both transmitting and receiving are transferred to this antenna the resulting system will be the most powerful in existence.

Novel antennas are employed at Arecibo and at the M.I.T. Haystack radar, shown in figure 2. The latter is completely enclosed in a radome made of metal struts supporting fiberglass panels. By protecting the instrument from wind, snow loading, and temperature changes, the engineers have been able to produce a large antenna capable of working to a wave length of 1 cm. The antenna system employed at Arecibo is a fixed spherical mirror supported in a vertically directed position. The feed system consists of a 96-foot section of wave-guide with slots cut in its sides, which acts to compensate for the spherical aberration. By displacing the feed from the zenith, the beam can be steered over a cone of  $40^\circ$ . Because the instrument is sited at  $+19^\circ$ N. latitude, planets are visible only when their declinations are in the range  $-1$  to  $+39^\circ$ . This restriction means that there are times in each year when one or more of the terrestrial planets cannot be viewed from Arecibo. The parameters listed in table 1 are somewhat optimistic because they do not include all the losses in each system. On the other hand, improvements are constantly being made. When the Arecibo system achieves full antenna gain, the



FIG. 2. An artist's sketch of the M.I.T. Haystack radar system, which employs a 120-foot precision parabola mounted inside a 150-foot space frame radome. This antenna operates as a radar at a wave length of 3.8 cm. The radar equipment is housed in a box that is hoisted into a position immediately behind the reflector apex.

TABLE 2  
DELAY DEPTH AND DOPPLER SPREAD FOR  
RADAR ASTRONOMY TARGETS

Target	Two-way delay depth to limb (msec.)	Limb-to-limb Doppler spread	
		Maximum (c./s.)	Minimum (1,000 Mc./s.)
Mercury	16.2	~43	~18
Venus	40.6	44	13.6
Mars	22.6	3,200	
Jupiter	~475	167,000	

system sensitivity will be improved by 6 to 8 db. The Cornell University group has developed plans for resurfacing the antenna for operation at a wave length of 10 cm., which would make an even larger improvement in the system performance.

#### E. THE SPREAD OF THE ECHO IN DELAY AND FREQUENCY

If an infinitesimally short transmitter pulse is reflected by one of the planets, it will be lengthened by the distribution of the scattering centers in delay. Similarly, a continuous-wave transmission will be broadened in frequency, because of the rotation of the planet. The limb-to-limb Doppler spread encountered at 1,000 Mc./s. and the two-way delay depth are listed in table 2.

There are two components that control the apparent rotation rate of a planet, namely, the intrinsic axial rotation, and the apparent rotation due to the motion of the planet past the earth. For Mercury and Venus these two components are of comparable magnitude, whereas for the other planets the broadening due to the intrinsic rotation is by far the largest. Mercury rotates about its axis in a direct sense (i.e., in the same direction as its motion in orbit). Since it overtakes the earth on the inside, the two components add to give the maximum Doppler spread at inferior conjunction (assuming that the spin axis is nearly perpendicular to the orbital plane). In the case of Venus the reverse is true because the intrinsic rotation is retrograde (Sec. VI).

Earlier when discussing the radar cross section, it was presumed that the whole planet is illuminated and contributing to the reflected energy. Thus, if pulses are employed, it is necessary that they be longer than the two-way radar depth listed in table 2. If pulses shorter than this are used, the echo power will fall and this effect is sometimes

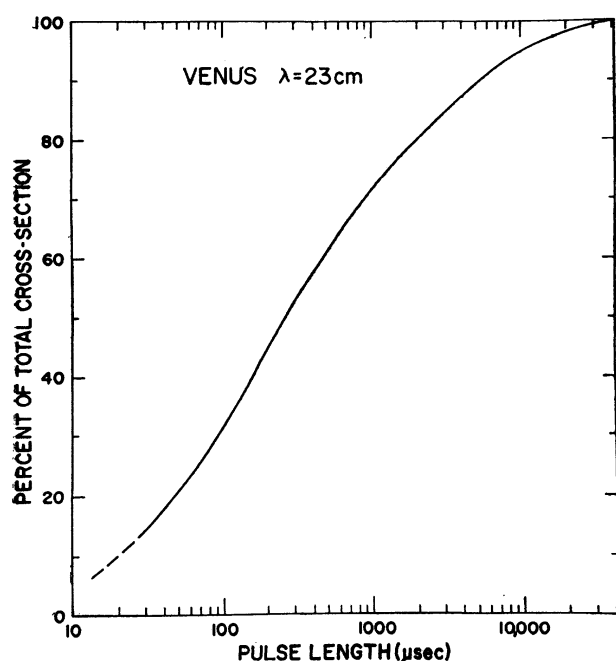


FIG. 3. The law showing the fraction of the total radar cross section of Venus obtained by pulses of different lengths. The total radar depth is 40.6 msec. (table 2), and if pulses less than this are employed the cross section will be reduced.

termed modulation loss. Figure 3 shows the modulation loss law observed for Venus at a frequency of 1,295 Mc./s. It can be seen that the effect is not very serious until a pulse having a depth of about 1/10 the planetary depth is reached. This is because the largest part of the energy reflected back to earth comes from a small region at the center of the disk where the surface is nearly normal to the ray path. Equally, if the bandwidth of the receiver is narrower than the value required to accept the limb-to-limb Doppler width of the signals, some of the echo power will be excluded, but here again a severe loss is not encountered until the bandwidth is reduced to about 1/10 the total limb-to-limb Doppler spread.

It should now be evident that the signal-to-noise ratio available at the receiver *output* depends upon (a) the radar parameters included in Eqs. (1) and (2), (b) the delay and frequency dispersion characteristics of the target (table 2), and (c) the manner in which the transmitted signals are modulated and demodulated. Without attempting to go into further details, we simply state that in experiments to measure the precise distance to the near face of the planet by employing short pulses (or some equivalent form of coded signal) there is an inevitable reduction in the cross section.

Thus, there is often a large difference between the distance at which a given radar will first *detect* a planet and the distance at which accurate range measurements can be carried out.

### III. MEASUREMENTS OF RANGE AND DOPPLER SHIFT

#### A. INTRODUCTION

We have discussed at some length the factors governing our ability to detect radio reflections from the planets. We have not considered in any great detail the actual engineering design of the equipment employed, believing that this would be out of place in a review of this kind. It should be borne in mind, however, that the nature of equipment available tends to determine both what planets can be seen and also what type of observations can be made.

In order to make any type of observation, it is first necessary to prepare an ephemeris giving tabular values of the azimuth, elevation, range, Doppler shift, etc., as functions of time. For the earliest observations these were generated from astronomical tables that list the heliocentric coordinates of the earth and the geocentric coordinates of the planet as a function of date (at four-day intervals for Venus and two-day intervals for Mercury) (Pettengill and Price, 1961). It was then necessary to interpolate to obtain these angles at, say, one-minute intervals. In order to fix the size of the triangle that these angles define, one must adopt a trial value of the Astronomical Unit. It was also necessary to assume a value for the planetary radius, since this affects the time delay (though not the Doppler shift). Following the first successful detection of echoes from Venus (Victor and Stevens, 1961; Staff of Millstone Radar, 1961; Thompson *et al.*, 1961; Kotel'nikov *et al.*, 1961), the accuracy achieved in the radar time delay measurements rapidly approached one part in  $10^8$ , requiring that these computations be carried out to still higher accuracy.

The precision to which Doppler-shift measurements can be carried out is fundamentally limited by the stability of the oscillators governing the operation of the radar system. At present, stabilities of a few parts in  $10^{11}$  are routinely obtained. Since in interplanetary measurement the Doppler shift never exceeds about one part in  $10^4$  of the transmitted frequency, the accuracy of Doppler shift determinations does not exceed about one part in  $10^7$  and may become very small at close

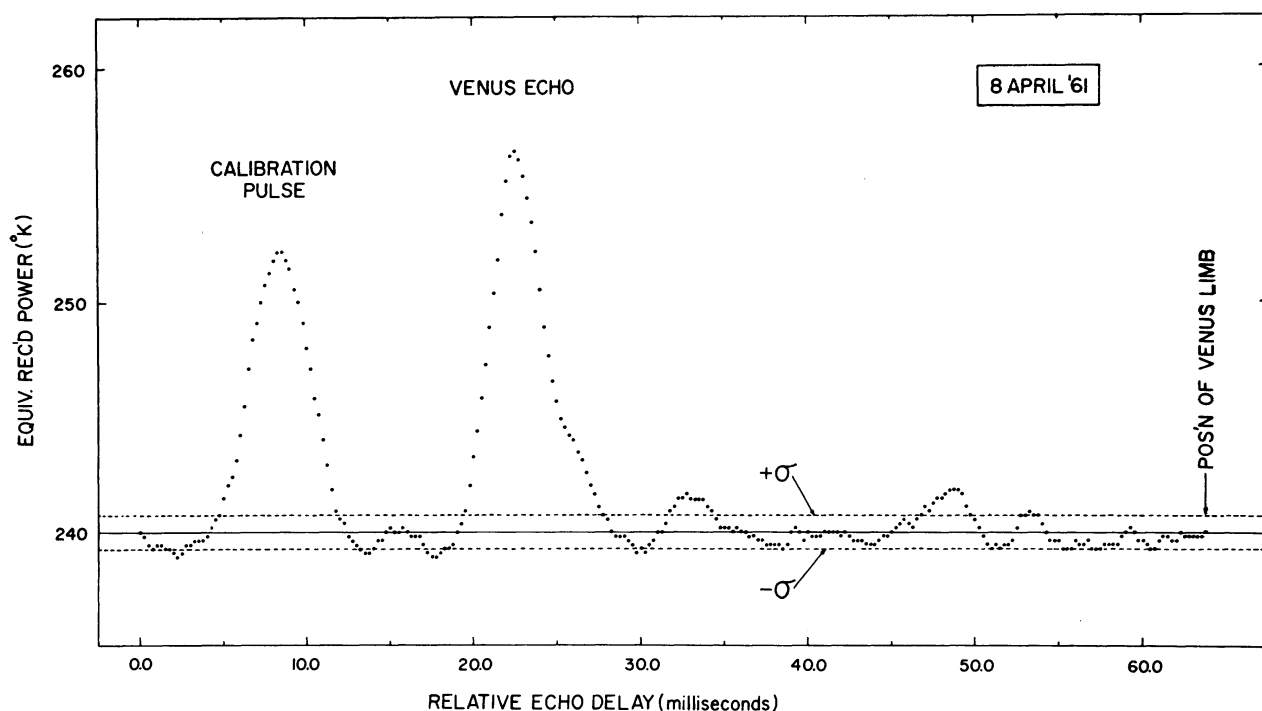


FIG. 4. A plot of incoherently summed power *vs.* delay over the 60-msec. interval between transmitter pulses. A calibration pulse giving a  $12^{\circ}$  K increase in the system temperature was inserted into the receiver at the commencement of each sweep. The echo from Venus is the second (larger pulse). These measurements were made with the Millstone radar in 1961 using 4-msec. pulses (Pettengill *et al.*, 1962).

approach when the Doppler shift goes through zero. Nevertheless, the usual first-order expression for the Doppler shift  $\Delta f = -2v/\lambda$ , where  $v$  is the relative velocity of the reflecting surface away from the observer, is not adequate, and a second-order term must be included. With the accuracy achievable in interplanetary Doppler measurements, this second-order term has been verified by a number of groups. The intrinsic time-keeping capability obtained by employing clocks driven by these fundamental frequency sources is such that the measurement of time delay is not yet limited by the stability of the oscillators. Thus, for the most part, range measurements are more useful for testing ephemerides, and in what follows we shall primarily discuss these.

Radar ranging experiments employing pulse transmissions (as at Arecibo) are conceptually the simplest to understand. In these measurements a train of equal-length, equi-spaced pulses is transmitted usually for a period long enough to allow the first pulse to travel to the planet and back. The transmitter is then turned off. On reception the signals are rectified and converted to digital numbers which are summed in a digital computer. The anticipated Doppler shift of the echoes is

compensated in the receiver by varying the tuning. Usually this is accomplished by causing one of the local oscillator frequencies to change with time in a way that matches the computed values. The device performing this operation may also control the digital sampling of the signals in such a way that the motion of the echo along the timebase (due to the constantly changing range) is also compensated.

Figure 4 shows the results of one of the early observations carried out at the Millstone Hill Radar (table 1) of Venus (Pettengill *et al.*, 1962). The plot shows the entire 60-msec. interval between transmitted pulses—usually termed timebase. By igniting a gas discharge tube coupled to the receiver input terminals on each sweep of the timebase, it is possible to include in the processing a pulse of known amplitude. This serves to calibrate the system temperature and echo power, and hence makes it possible to obtain a precise measurement of the cross section  $\sigma$ . The computed standard deviation of the echo power based upon the known number of samples that have been included in the integration process is also shown. A 4-msec. pulse was employed in these measurements and the samples of the output

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PULSE = 4 Msec  
 PRF = 8PPS  
 POWER = 100Kw

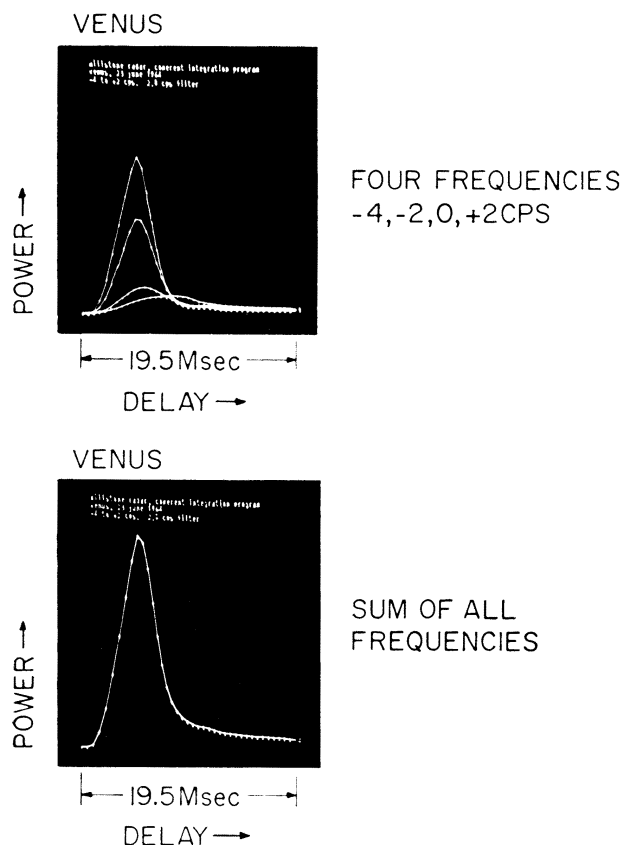


FIG. 5. These photographs are of oscilloscope displays obtained after coherently processing the echoes in a digital computer. The upper picture shows the echo from Venus observed in four predetector filters, each 2 c.p.s. wide, and tuned to frequencies of  $-4$ ,  $-2$ ,  $0$  and  $+2$  c.p.s. with respect to the expected frequency. The strongest echo is observed in the  $-2$  c.p.s. filter. The lower picture shows the sum of the power in all four filters.

power of the receiver were summed incoherently, i.e., without regard to the phase of the signals. In this process the uncertainty in the mean output power varies as  $1/\sqrt{n}$ , where  $n$  is the number of sweeps summed.

The signals may be coherently detected using sine and cosine phase detectors and then added coherently (i.e., with regard to phase) in the computer. This scheme makes use of the phase coherence that exists between groups of pulses (due to the small amount of spectral broadening introduced by the slow rotation of the planet).

Thus, if phase coherence is found to persist over  $N$  successive echoes, one can achieve a  $\sqrt{N}$  improvement over incoherent processing. Figure 5 shows the results of processing 4-msec. pulses coherently in a computer (Evans *et al.*, 1965). The upper picture shows the echo from Venus observed in four predetector filters, each 2 Hz. wide and tuned to frequencies of  $-4$ ,  $-2$ ,  $0$  and  $+2$  Hz. with respect to the expected frequency. The strongest echo is observed in the  $-2$  Hz. filter. The lower picture shows the sum of all four filters. By coherently processing, one can therefore determine simultaneously the range and the Doppler shift of the echoes.

In order to improve the range resolution, it is necessary to reduce the pulse length, and often it is more convenient instead to code the pulse in a way which can be recognized on reception. There is a variety of possible coding schemes, and one which has been widely used is to reverse the phase in the pulse by  $180^\circ$  according to a pattern which, when cross-correlated with itself, yields one central peak and low sidelobes. That is, the transmitted pulse (lasting  $T$  seconds) is of constant amplitude and frequency but has its phase reversed according to a pseudo-random code. The minimum interval between code reversals—the so-called baud length—is fixed as  $T'$  seconds ( $T' \ll T$ ), and all phase reversals occur at multiples of  $T'$  seconds. Usually the received signals are sampled at intervals of  $T'/2$  seconds, and the range resolution achieved with this scheme has been found to be the order of  $T'/5$  seconds when the signals are strong.

The decoding of these pulses on reception can be performed in a variety of ways. At Millstone it was accomplished by connecting the receiver to a tapped filter delay line that provides an over-all delay  $T$  equal in length to the transmitted pulse. The taps are connected to a summing point via  $0^\circ$  or  $180^\circ$  phase shifters arranged according to the same pattern as was transmitted. The output from the summing point is connected to the sine and cosine phase detectors as before. A system of this type that provides a pulse compression of 11 times was used to produce compressed pulses effectively 360 and 40  $\mu$ sec. long (Evans *et al.*, 1965).

At Arecibo a computer has been employed to perform the operation of decoding the pulses as well as spectrum analysis and integration. In this arrangement, the computer stores the transmitted pattern and then inverts the phases of the data samples according to the same pseudo-random



code and adds a sequence of samples occupying the same time interval as the transmitted pulse ( $T$  sec.) to yield the equivalent of echo samples taken with a very short pulse. With this arrangement it is possible to allow the pulse length  $T$  to grow, while preserving range resolution (set by the baud length  $T'$ ) fixed. Eventually a point is reached when the ends of the pulses are touching, i.e., the transmitter is radiating continuously. This is the method in which the CW transmitters in the Goldstone and Haystack radars are modulated. At Haystack, values of  $T' = 60 \mu\text{sec.}$  and  $24 \mu\text{sec.}$  are routinely used. The computer performs the decoding during the time the signals are being received and then performs the tasks of frequency analysis and integration during the transmitting period that follows (Shapiro *et al.*, 1968).

At the Jet Propulsion Laboratory the decoding has been accomplished by modulating the phase of the first local oscillator in the receiver by  $\pm 180^\circ$  according to the same code after a predetermined delay (Tausworthe, 1965). It is then necessary to have separate receiver chains for each delay that is to be examined.

Since the echo power has limited frequency extent, the optimum reception is achieved when the bandwidth at the predetector filter (which may in practice be synthesized by the coherent processing carried out in the computer) is adjusted to match the width of the echo power. At any instant the short pulse illuminates an annulus on the planetary disk as shown in figure 6. Regions yielding the largest Doppler shift are those parts of the annulus lying farthest from the projected spin axis. Evidently the Doppler broadening increases as the pulse travels over the surface. Thus, any single filter would be matched to the echo bandwidth at only a specific delay. Under these circumstances, it can be shown that the optimum processing procedure is to determine the echo power *vs.* frequency at a large number of delays, weight the entire array of power *vs.* delay and frequency according to the amount of power *expected* in the absence of noise and sum. This requires that the echo signature be known *a priori*, e.g., from observations when the planet may have been closer. By moving the template both with respect to delay and Doppler shift and repeating the summing operation, a plot of total weighted echo power *vs.* trial position is obtained. The position at which this sum maximizes is the best estimate of delay and Doppler shift (Price, 1968).

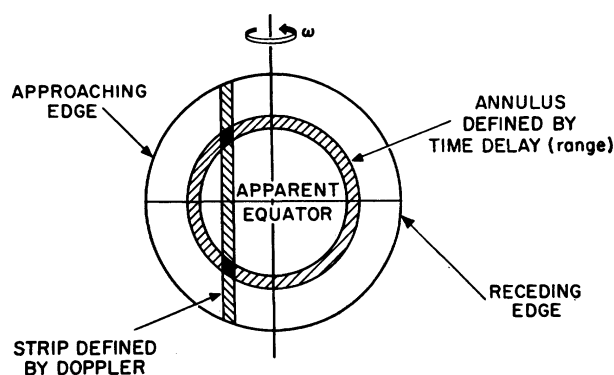


FIG. 6. Diagram of the disk of a planet as seen from earth. By transmitting a short pulse it is possible to resolve a narrow annulus as shown. Alternatively, if CW transmissions are employed, it is possible by spectrum analysis of the echoes to select strips parallel to the projected axis of rotation. By combining the two techniques (delay-Doppler mapping) it is possible to isolate the small heavily shaded regions.

This refined type of matched-filter processing is routinely conducted at Haystack and contributes significantly to the sensitivity and therefore the range accuracy of the system.

#### B. OBSERVATIONS OF VENUS

Radar echoes from Venus were first reliably detected in 1961 by groups in the United States, Great Britain, and the U.S.S.R. during a period around inferior conjunction which occurred on April 10. Observations have continued at each succeeding conjunction. The history of this early work has been summarized by Pettengill (1968), and the values of the Astronomical Unit derived from these measurements (which agreed well amongst themselves) by Shapiro (1965, 1968). In many respects the first detection proved the most difficult. Not merely was the equipment in a far less advanced state than at present, but the uncertainty in the value of the Astronomical Unit caused large errors to be made in the "trial" values adopted by the various groups for the computation of the initial ephemerides. For the M.I.T. group, the uncertainty in the Astronomical Unit introduced an uncertainty in the flight time that was larger than the separation  $t$  between pulses (Pettengill *et al.*, 1962). Thus, when the echo was recognized on the timebase (following approximately five minutes integration) it was known only that the total delay  $\tau = mt + \Delta t$ , where  $\Delta t$  is the (measured) delay of the echo along the timebase. The value of  $m$ , i.e., the number of pulses "in flight" was unknown. This difficulty was resolved

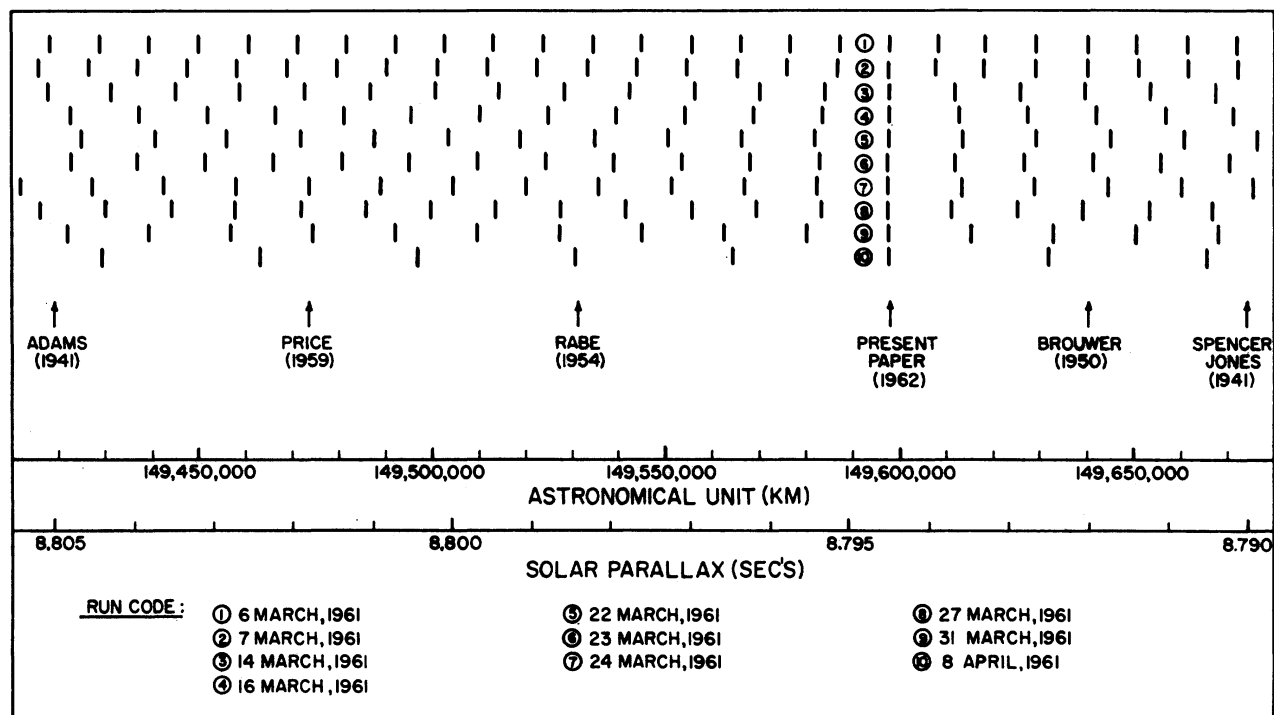


FIG. 7. Diagram showing method of resolving the periodic uncertainty in the A. U. which results when a simple pulse train is used, by varying the radar interpulse interval and target distance (after Pettengill *et al.*, 1962).

by making observations with different values of  $t$  and solving for a number of likely values of the A.U. (i.e., different  $m$ ) for each case. The true value can then be recognized as the only one common to all experiments (fig. 7). Figure 7 usefully illustrates how large the difference is between the radar value of the Astronomical Unit and earlier values obtained from optical astronomy (Sec. V) or the value adopted at the time by the International Astronomical Union (defined as solar parallax = 8.800 arc sec.).

The early Venus observations were carried out at a time when it was widely thought that the orbital elements of the near planets were well known and hence radar determination of the distance to any one would determine the size of the system as a whole. This notion was quickly dispelled, however, by the discovery that no single value of the A.U. could be adopted which correctly predicted the flight time of the pulses observed in all the measurements (Muhleman *et al.*, 1962; Pettengill *et al.*, 1962). The nature of this discrepancy is shown in figure 8, where residuals between the observed and predicted flight times are shown for observations made at Millstone in 1964 (Evans *et al.*, 1965). Initially, Venus was found to be about 300 km. in advance of the pre-

dicted position, whilst at the termination of the observations it was approximately the same distance behind. This indicates an error in the assumed position of Venus in its orbit (defined by the initial mean anomaly) amounting to approximately 0.5" arc at the sun (Muhleman *et al.*, 1962). A second variation can be seen in figure 8 as an oscillation in the residuals with a period of about 30 days. This proved to be introduced by an error in the adopted value of the earth-moon mass ratio (Ash *et al.*, 1967). As a result of these discrepancies, further efforts to refine the Astronomical Unit by radar have required a simultaneous improvement in the elements of the orbits of the planets under observation (Secs. IV and V).

Currently, range determinations of Venus are being undertaken chiefly at Arecibo and Haystack where the radar sensitivity is adequate to follow the planet round its entire orbit and determine the flight time at all times to an accuracy of about  $\pm 10$   $\mu$ sec. or better (Pettengill *et al.*, 1967; Shapiro *et al.*, 1968).

#### C. OBSERVATIONS OF MERCURY AND MARS

Radar detection of Mercury was first accomplished in 1962 (Kotel'nikov, 1962) and yielded an approximate value for the A.U. in support of

the earlier radar determination using Venus. Similar results were obtained by the JPL group in 1963 (Carpenter and Goldstein, 1963). Later observations at Arecibo (Pettengill *et al.*, 1967) served to show that even larger residuals are encountered for Mercury than Venus (Ash *et al.*, 1967).

Detection of Mars was first accomplished during the opposition of February 1964 (Goldstein and Gillmore, 1963; Kotelnikov *et al.*, 1963a). Because Mars is a more difficult target to observe than either Mercury or Venus, subsequent observations have centered around the times of opposition (March, 1965, and April, 1967). Nevertheless, they have been quite useful for redetermining the elements of the Mars orbit (M. E. Ash, I. I. Shapiro, and W. B. Smith, private communication)—many of which needed corrections about as large as those encountered for Venus. One complicating factor for Mars that appears largely ab-

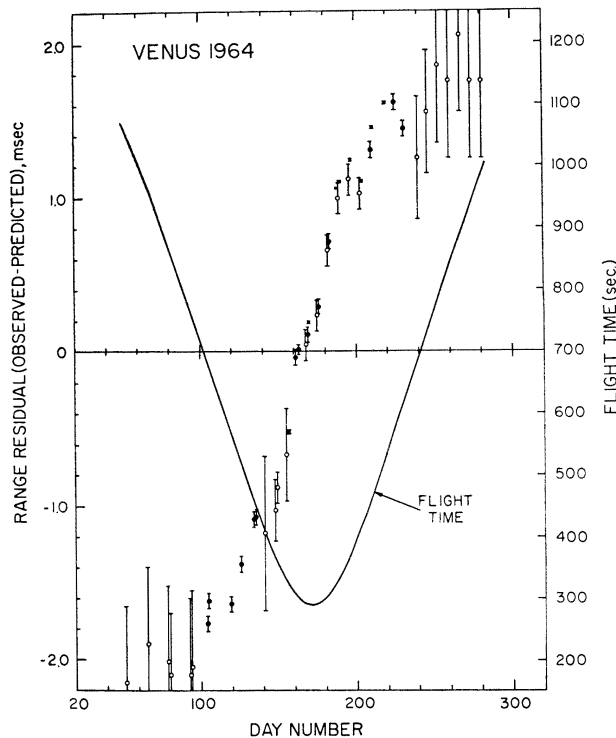


FIG. 8. The Venus range residuals (observed-predicted) measured at Millstone Hill during a 7-month period centered upon the inferior conjunction (June 20, i.e., day number 172) in 1964. The variation of the error bars with date reflects the change in the pulse length that had to be made in order to assure detection at the greater ranges. The S-shape character of the residuals indicates that Venus is ahead of its predicted position with respect to the earth by about  $0.5''$  heliocentric arc.

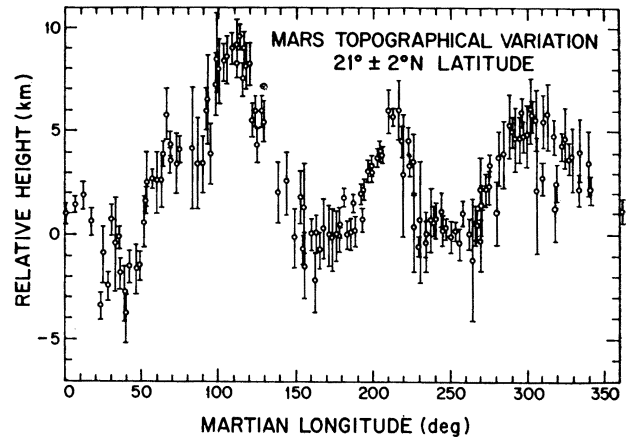


FIG. 9. Variation of the height of the Martian surface with respect to an arbitrary mean radius for  $21^\circ \pm 2^\circ$  N. latitude. These measurements were made by precision ranging using the Haystack radar (Pettengill *et al.*, 1969).

sent for Venus and Mercury is the existence of significant topographical height variations. Because of the rapid axial rotation these can be separated readily from variations in distance due to orbital motion. Figure 9 gives height variations of the surface (relative to some as yet ill-defined mean radius) obtained at Haystack in 1967 (Pettengill *et al.*, 1969).

#### D. OBSERVATIONS OF ICARUS

The asteroid Icarus passed the earth at a distance of  $6.5 \times 10^6$  km. on June 14, 1968, and was successfully detected at Haystack and Goldstone (Goldstein, 1968). A prerequisite for this detection was the computation of an accurate ephemeris and this was undertaken by a number of separate groups who first redetermined the orbit of Icarus by obtaining data from all earlier sightings. Photographic observations of Icarus immediately before the June 14 encounter permitted last-minute recomputation of the orbit and radar ephemerides. In both experiments CW signals were transmitted, yielding only estimates of the Doppler shift and cross section. Figure 10 shows the signal spectrum obtained at Haystack with the echo from Icarus close to the predicted frequency. It appears that the trial ephemeris was remarkably accurate and the object has a diameter of the order of 0.5 km. (Goldstein, 1968; G. H. Pettengill, private communication).

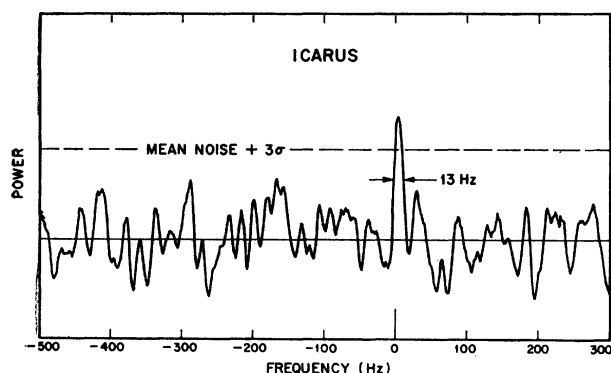


FIG. 10. Spectrum of signals obtained during observations of Icarus in 1968. The echo is weak but clearly recognizable (G. H. Pettengill, private communication).

#### IV. THEORETICAL PREDICTIONS

The conceptual and computational difficulties associated with making accurate delay and Doppler predictions have been reviewed by Shapiro (1968). Based upon past observation, we assume that to first order the motions of the planets with respect to an inertial coordinate system are in accord with Newton's laws. Current practice is to adopt a coordinate system whose origin is at the center of mass of the solar system, and whose axes are fixed with respect to the stars (after allowing for proper motions). Newton's laws enable one to calculate with arbitrary accuracy the future position of the planets provided only that their positions and velocities at some precise epoch are known. Newcomb (1895) reviewed a very large amount of earlier optical data in an analytical perturbation theory in order to arrive at what are essentially a set of accurate initial conditions which are used to this day as the basis of the standard astronomical tables, e.g., "The American Ephemeris and Nautical Almanac."

Newton's laws relate positions and velocities with respect to time. Until quite recently the rotation of the earth was accepted as the best unit of time. Since the advent of precision crystal and atomic clocks, periodic variations in the length of the sidereal day long suspected from astronomical observations have been confirmed. Accordingly, astronomers introduced a unit of time (Ephemeris Time) defined by the earth's orbital motion and hence based upon Newcomb's theory of the motion of the sun as seen from the earth. [In practice, the variation of Ephemeris Time with respect to Universal Time (determined from the earth's ro-

tation) is obtained from observations of the moon and hence depends upon Brown's lunar theory.]

In the system of equations, the unit of mass was initially chosen to be that of the sun and the unit of distance, the semi-major axis  $a$  of the earth's orbit (A.U.). This unit is related to the others via Kepler's third law, which states

$$a^3 \propto (kP)^2 \quad (4)$$

where  $P$  is the orbital period, and  $k$  the gaussian constant which involves the mass of the sun  $M_s$ , and the gravitational constant  $G$ . The orbital period  $P$  is defined in units of time (earlier the mean solar day, but now defined as 86,400 ephemeris seconds). Thus, when  $M_s$  is set equal to unity, the unit of length  $a$  is specified in terms of  $G$ . Since this constant is used in many computations, astronomers have chosen to maintain its value fixed, with the result that later improvements have caused the value of the astronomical unit  $a$  so defined to be no longer precisely the same as the semi-major axis of the earth's orbit.

Given the above system of units together with the initial conditions, it is possible to integrate the equations of motion to obtain the positions of the planets as functions of time. The first-order terms of these motions computed according to Einstein's theory are the same as those of Newton's theory, so that one can obtain sufficient accuracy in predicting the motions according to general relativity by adding a correction to the motion predicted from Newtonian theory. The largest effect is on the advance of the perihelia of the planetary orbits, which in the case of Mercury amounts to 43 arc sec. per century in excess of that predicted by Newtonian theory.

In order to convert the planetary distances computed from these equations into predictions of the flight time for a short pulse, it is necessary to adopt a value for the A.U. in terms of light seconds. [Presently this is known with greater precision than the velocity of light and thus constitutes a better definition than a value of the A.U. in terms of km.] In general, the distance between the observer and the planetary surface is changing with time, and thus the flight time for a pulse cannot be calculated in closed form but requires iterative calculations using a digital computer. The positions of the observer at the instant of transmission and the instant of reception may be taken as the foci of an ellipsoid of revolution. The surface of the ellipsoid encompasses all possible positions that the reflection point may occupy and

be at a delay  $\tau$ . Thus, we seek that member of the family of ellipsoids of resolution to which the near surface of the planet is tangent at the instant of reflection. There is only one such ellipsoid and hence a unique solution for  $\tau$ . The round trip Doppler shift is given by  $\Delta f = -f\dot{\tau}$  where  $f$  is the operating frequency of the radar and all times and frequencies are as measured by the observer at the time of reception.

To include the effects of general relativity on predictions of flight time leads to interesting effects for ray paths that pass close to the sun, since for such paths the influence of the solar gravitational field on the velocity of light becomes important (Shapiro, 1964; 1966). These effects lead to the possibility of a fourth test of general relativity. The magnitude of the effect is illustrated in figure 11 in the case of Mercury. It can be seen that near inferior conjunction, general relativity effects increase the flight time by  $\sim 15 \mu\text{sec.}$ , and were this constant it would be indistinguishable from a 2-km. decrease in the planetary radius. However, after elongation is reached, the excess delay increases rapidly and reaches a value of  $\sim 190 \mu\text{sec.}$  for a ray grazing the surface of the sun. In order to distinguish this effect from additional delays introduced by the solar corona, it is necessary to employ a high operating frequency  $f$ , which makes Haystack the existing radar best suited for this measurement (table 1). It is also necessary, of course, to determine accurately the orbit of Mercury from observations (near inferior conjunction) that are distributed around the orbit.

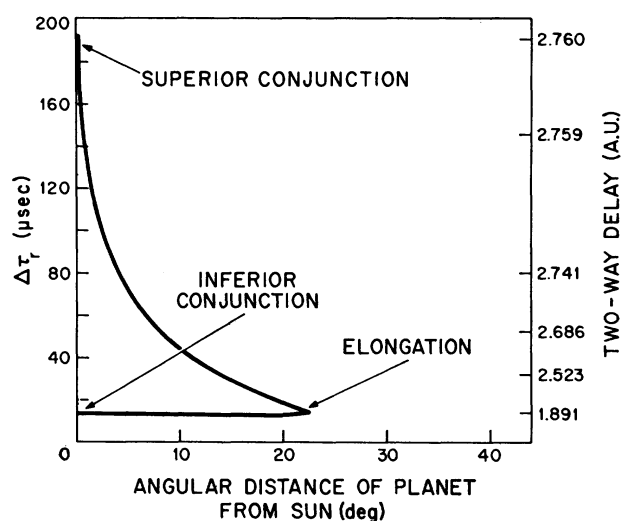


FIG. 11. The excess delay  $\Delta\tau$ , computed for a pulse to travel to Mercury arising from effects of general relativity (Shapiro, 1964).

## V. RESULTS FOR PLANETARY ORBITAL MOTIONS

### A. METHOD OF REFINEMENT AND COMPARISON WITH THEORY

Data from planetary radar observations have been rapidly accumulated since 1961. The residuals observed between *a priori* predictions based upon existing astronomical tables and the measurements have been large enough to conclude that some of the elements of the orbits of the terrestrial planets are in need of revision. This can best be accomplished by recomputing the predicted

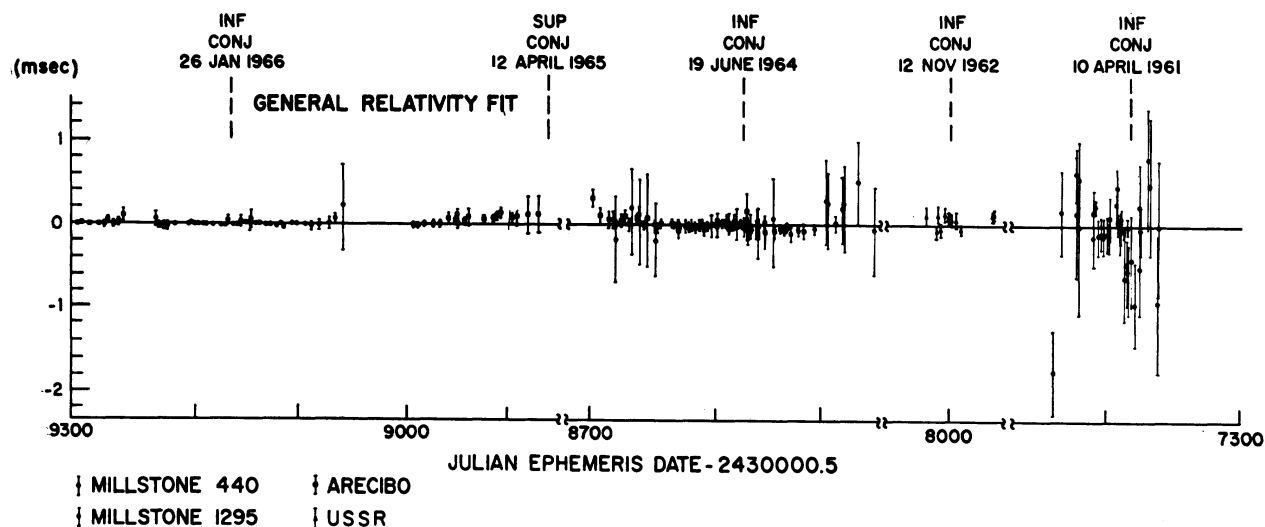


FIG. 12. Residuals of the radar flight time data following adjustment of the A. U. and elements of the orbits of earth and Venus (Ash *et al.*, 1967).

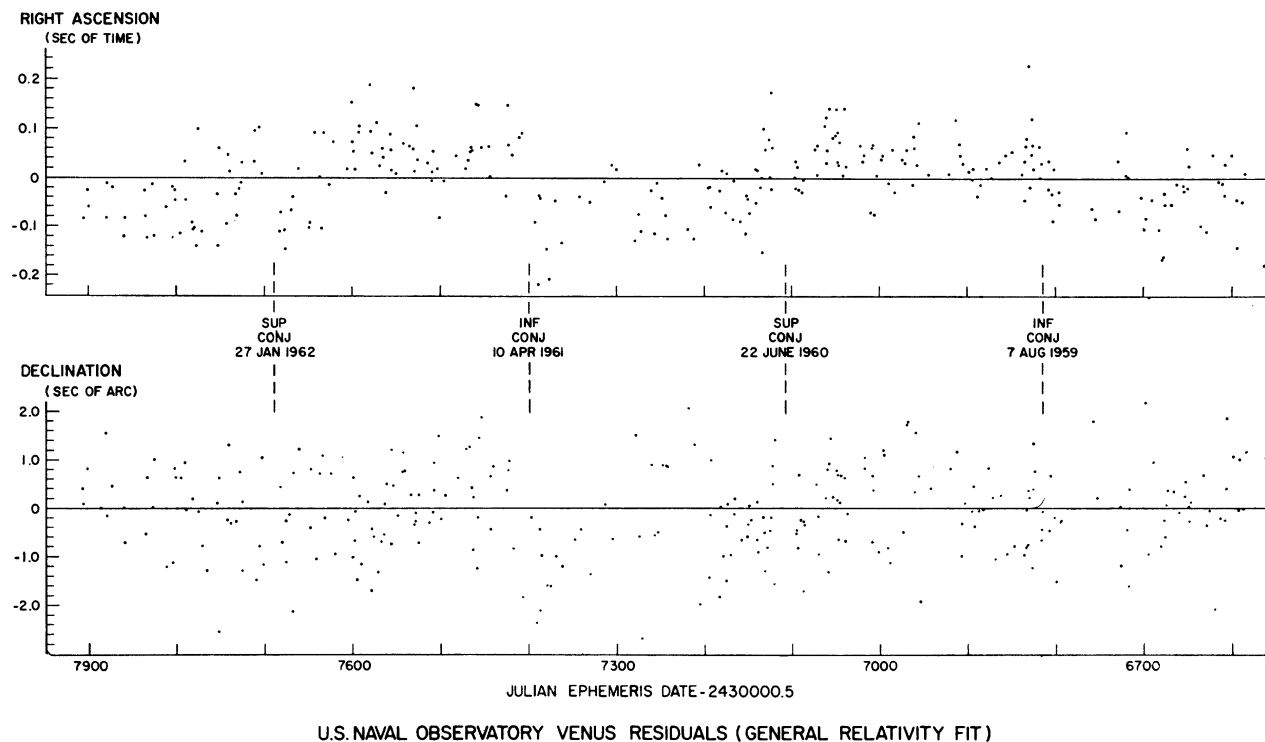


FIG. 13. Sample of residuals in the right ascension and declination of Venus from meridian circle observations for the general relativity solution. Note the systematic errors in the right-ascension residuals which indicate that the limb-to-center corrections for Venus should be modified (after Ash *et al.*, 1967).

delays (and Doppler shifts) until the predictions agree in a least mean squares sense with the observations. The trial values of the elements are determined by noting the way in which the deviations depend upon a small change in each element. Because there are abundant observations, i.e., the set is redundant, it is possible to vary a large number of parameters simultaneously without fear that the process will fail to converge or will converge on erroneous values. Despite this, it is desirable to determine in advance which of the elements are sensitive to the radar observations and to hold the others fixed and equal to the values obtained from optical observations.

An even better procedure adopted by Ash *et al.* (1967) is to include both optical and radar data and solve for best fit (i.e., a minimum in the mean square residual) to both sets simultaneously. In the work undertaken by the M.I.T. group (Ash *et al.*, 1967), the Arecibo and Millstone radar observations of Mercury and Venus together with the U. S. Naval Observatory meridian circle observations of these planets between 1950 and 1965 formed the data set. Assumed known were the heliocentric orbits and masses of Jupiter and the outer planets, and the geocentric orbit of the moon.

Allowed to be adjusted for best fit to the observations were the 6 orbital elements of earth, Venus, and Mercury, the masses and radii of Venus and Mercury, and the masses of Mars, earth and moon and the Astronomical Unit, giving a total of 26 parameters.<sup>1</sup> One solution was obtained assuming Newtonian laws to hold, and a second assuming the general relativity modified Newtonian system. This is necessary since the values of the parameters depend upon what system of physical laws one assumes to govern the planetary motions. A comparison of theories can be made by observing which best matches the observations. This can be accomplished by computing the sum of the squares of the normalized residuals  $(O_i - C_i)/\sigma_i$ , where  $O_i$  is the observed value,  $C_i$  is the post-fit computed value, and  $\sigma_i$  the uncertainty assigned to the measurement by the observer.

Figure 12 compares some of the residuals observed for the Venus radar data after the fitting procedure has been completed. It can be seen

<sup>1</sup> In the earliest reduction (Ash *et al.*, 1967), computational difficulties limited the solution to 23 of these parameters at any one time. However, by repeating the process several times for different sets, a convergent solution was obtained for all 26 parameters.

that, compared with prefit residuals (e.g., fig. 8), the agreement is very good and few of the measured points fall farther from the computed value than their associated uncertainty. Figure 13 shows residuals for the optical position of Venus. It can be seen that the right ascension residuals undergo an abrupt change of sign at inferior conjunction when the crescent moves from one side to the other. This suggests that the limb-to-center corrections employed by the Naval Observatory should be modified (Ash *et al.*, 1967).

#### B. THE VALUE OF THE ASTRONOMICAL UNIT

Figure 14 compares the value of the A.U. derived by radar observations of Venus in 1961 with earlier optical determinations. Of these only that published by Brouwer (1950) could be said to be consistent with the radar value and in large measure this is simply due to the considerable error associated with Brouwer's determination. The values published by Spencer Jones (1941) and Rabe (1954) both rested upon observations of the minor planet Eros. In 1931 the British astronomer Spencer Jones organized observations from which the distance to Eros could be deduced by triangulation. Rabe (1954) determined the A.U. from the gravitational perturbation produced on the orbit of Eros as a result of its passage close to the earth. Both methods yielded values of the A.U. with an uncertainty that was placed at 1 part in  $10^4$  yet differed by 1 part in  $10^3$ . The discovery of flexure in some of the photographic plates used in the triangulation determination served to discredit Spencer Jones' work with the result that Rabe's gravitational value was widely accepted as the best available value prior to the radar detection of Venus. In recent years, Rabe (1967) has discovered a conceptual error in his earlier work and re-evaluated the value of the A.U. from the perturbation of Eros, deriving a

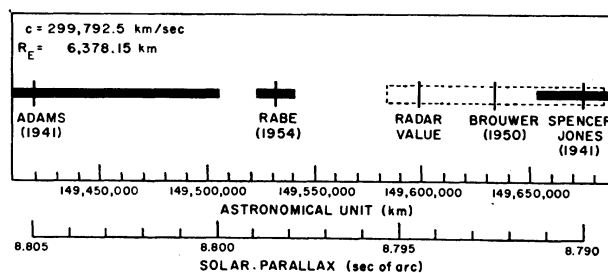
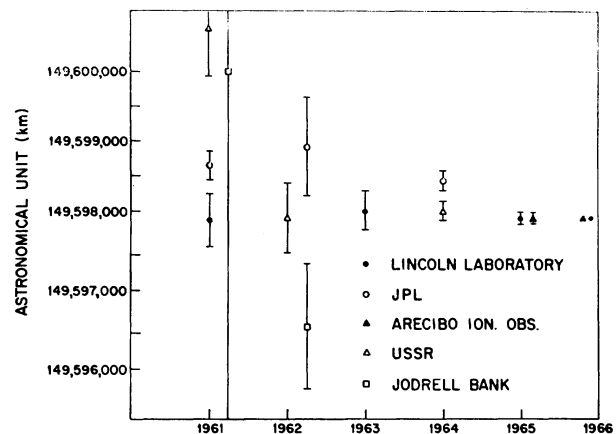


FIG. 14. Comparison of the radar value of the astronomical unit with earlier optical determinations.



RADAR DETERMINATIONS OF THE ASTRONOMICAL UNIT

FIG. 15. Values of the astronomical unit as determined by radar since 1961.

value in good agreement with the radar determination (Rabe and Francis, 1967).

The values reported for the A.U. from radar observations of Venus in 1961 were in remarkably good agreement, compared with earlier optical determinations, yet were in fact spread by more than the sums of the assigned probable errors (fig. 15). It seems that this resulted less from inaccuracy in the range determination than in converting these measures to a value of the A.U. In later years, improvements in radar ranging accuracy (Sec. III) have gone hand in hand with refinements of the orbital parameters (Sec. IV), with the result that the radar values appear to have converged as shown in figure 15. Expressed in light seconds, the value of the A.U. given by Ash *et al.* (1967) is

$$1 \text{ A.U.} = 499.004786 \text{ light sec.}$$

with a formal standard error of  $\pm 5$  in the last place. Were the velocity of light known with equal accuracy, this would correspond to an uncertainty of  $\pm 1$  km. It seems likely, however (as Ash *et al.* have pointed out), that as in earlier determinations, systematic errors exist that are larger than the formal standard error, yet it seems difficult to discover anything which would introduce errors as large as five times this amount. A reduction of the formal error below  $\pm 5 \mu\text{sec.}$  is readily possible, but at this level a number of known effects such as propagation delays in the atmosphere of the earth become significant, and it may be difficult to correct properly for each of these effects.

TABLE 3  
OSCULATING ELLIPTIC ORBITAL ELEMENTS AT JED 2439340.5 REFERRED TO  
MEAN EQUINOX AND EQUATOR OF 1950.0

Orbital element	Mercury		Venus		Earth-moon barycenter	
	Newtonian fit	Relativity fit	Newtonian fit	Relativity fit	Newtonian fit	Relativity fit
Semimajor axis $a$ (A.U.)	0.387098450436	0.387098414951	0.723329902805	0.723329859677	1.000003838146	1.000003794771
Eccentricity $e$	0.2056252912	0.2056252616	0.0067893379	0.0067893036	0.0166821339	0.0166821414
Inclination $i$ (deg.)	28.6032096	28.6032349	24.4665221	24.4665221	23.4435784	23.4435741
Right ascension of ascending node $\Omega$ (deg.)	10.8602455	10.8601939	7.9788463	7.9788228	0.0004331	0.0004358
Argument of perihelion $\omega$ (deg.)	66.9331816	66.933922	123.3335336	123.3336740	102.1909322	102.1909785
Initial mean anomaly $l_0$ (deg.)	269.8932013	269.8930787	297.4518134	297.4517291	208.7069508	208.7069368

C. VALUES FOR THE RADII AND MASSES  
OF MERCURY AND VENUS

According to Ash *et al.* (1967) the values for the reciprocal of the masses of Mercury and Venus (i.e.,  $M_{\text{sun}}/M_{\text{planet}}$ ) are 6,021,000 and 408,250, respectively, with formal errors of 53,000 and 120. The radii of these two planets are 2,434 km., and 6,050 km. (Ash *et al.*, 1968) where the formal errors are  $\pm 2$  and  $\pm 1$  km. (It must be borne in mind that these error estimates are based only upon the spread of the points and the actual errors may be much larger.) Adopting these values, Ash *et al.* (1967) find that Mercury is as dense as earth, and Venus about 5 per cent less dense.

The value for the radius of Venus derived by radar has recently assumed considerable importance. By combining tracking data and density measurements obtained by the U. S. probe Mariner V and the Soviet probe Venera IV, a value for the radius of  $\sim 6,080$  km. was obtained (Kliore *et al.*, 1967). If instead the radar value is accepted, one is forced to conclude that Venera IV had not reached the surface of Venus when it ceased transmitting. Thus, instead of a surface

pressure of 20 atmospheres and temperature of  $550^\circ$  K as reported by Soviet workers, the true values are probably more nearly 100 atmos. and  $750^\circ$  K (Kliore and Cain, 1968). The weight of present evidence favors this latter conclusion since the radar value of the radius seems quite inconsistent with a value of  $\sim 6,080$  km. (Ash *et al.*, 1968; Melbourne *et al.*, 1968), and the radio brightness properties of Venus and radar cross section variation favor the higher values of pressure and temperature (Kuzmin and Vetukhnovskaya, 1968; Evans and Ingalls, 1968; Wood *et al.*, 1968).

D. ORBITAL ELEMENTS

Ash *et al.* (1967) have published revised elements of the orbits of Mercury, Venus, and the earth-moon barycenter, both for Newtonian and general relativity solutions, based upon radar observations from 1959 to July, 1966, and optical data from 1950 to 1965. These results are summarized in tables 3 and 4. Since then, the data set has been extended by including radar observations taken since 1965 (Shapiro *et al.*,

TABLE 4  
FORMAL STANDARD ERRORS OF ORBITAL ELEMENT ESTIMATES

	Mercury			Venus			Earth-moon barycenter		
	Radar and optical	Radar	Optical	Radar and optical	Radar	Optical	Radar and optical	Radar	Optical
$a$ (A.U.)	$1.0 \times 10^{-9}$	$1.2 \times 10^{-9}$	$1.4 \times 10^{-8}$	$2.1 \times 10^{-9}$	$3.1 \times 10^{-9}$	$3.5 \times 10^{-8}$	$7.4 \times 10^{-9}$	$9.1 \times 10^{-9}$	$1.3 \times 10^{-7}$
$e$	$2.2 \times 10^{-8}$	$2.3 \times 10^{-8}$	$7.3 \times 10^{-7}$	$1.7 \times 10^{-8}$	$1.8 \times 10^{-8}$	$3.4 \times 10^{-7}$	$1.9 \times 10^{-8}$	$2.1 \times 10^{-8}$	$1.8 \times 10^{-7}$
$i$	0".04	6".5	0".16	0".02	6".5	0".05	0".02	6".5	0".02
$\Omega$	0".10	27".0	0".31	0".08	31".7	0".12	0".07	33".1	0".07
$\omega$	0".10	27".4	0".83	0".40	31".7	12".2	0".14	33".1	2".4
$l_0$	0".03	0".03	0".86	0".40	0".43	12".2	0".12	0".13	2".4



1968). These later refinements have shown significant differences between the Newtonian and general relativity solutions (next section).

It is of interest to note in table 4 that the inclusion of the radar data has improved our knowledge of the eccentricities  $e$  and initial mean anomalies  $l_0$  of Mercury and Venus, by more than an order of magnitude. The inclination,  $i$ ; right ascension of the ascending node,  $\Omega$ ; and argument of perihelion,  $\omega$ , are improved but by lesser amounts. When these last three elements are determined using the radar data alone (table

4), the accuracy is far inferior to that of the optical data alone. In sum, the optical observations largely control the values of  $i$  and  $\Omega$ , whilst radar observations have chiefly determined  $a$ ,  $e$ , and  $l_0$ . Here we see the complementary nature of the two observing techniques that was claimed earlier.

#### E. GENERAL RELATIVITY

As noted in Section IV, the general theory of relativity predicts two effects not predicted by Newtonian theory. The first is an additional increase in the perihelion advance of the planets, and the second a slowing down of pulses that travel past the sun. An examination of the data has been made to search for this second effect (Shapiro *et al.*, 1968). In this work, a solution for the elements was obtained including the first (i.e., perihelion advance) but not the second prediction of general relativity. The residuals for two superior conjunctions of Mercury were then plotted and compared with the additional time delay expected because of the effect of the solar gravitational field on the time of flight of the pulses. Figure 16 shows the results. It seems clear that this Fourth Test of relativity does indeed support Einstein's general theory, but at the present level of accuracy of the measurements it is impossible to distinguish between this and predictions based upon the Brans-Dicke theory (Brans and Dicke, 1961) where the free parameter has been set in accord with the estimate of the solar oblateness of Dicke and Goldenberg (1967). Nor is it possible to place limits upon the quadrupole moment of the sun, though it is anticipated that as the measurements continue these may become possible.

### VI. PLANETARY ROTATION RATES

#### A. METHODS OF DETERMINATION

There are basically three methods by which the rotation of Venus has been determined by radar. Historically, the first is the so-called "base-band-width" technique in which the Doppler broadening of a CW signal is examined. The signals scattered from points on the limb lying farthest from the apparent spin axis suffer a Doppler shift

$$f_o = \pm \frac{2\omega_T r}{\lambda} \quad (5)$$

with respect to signals reflected from the center, where  $\omega_T$  is the apparent angular rotation rate,  $r$  the planetary radius, and  $\lambda$  the wave length. If

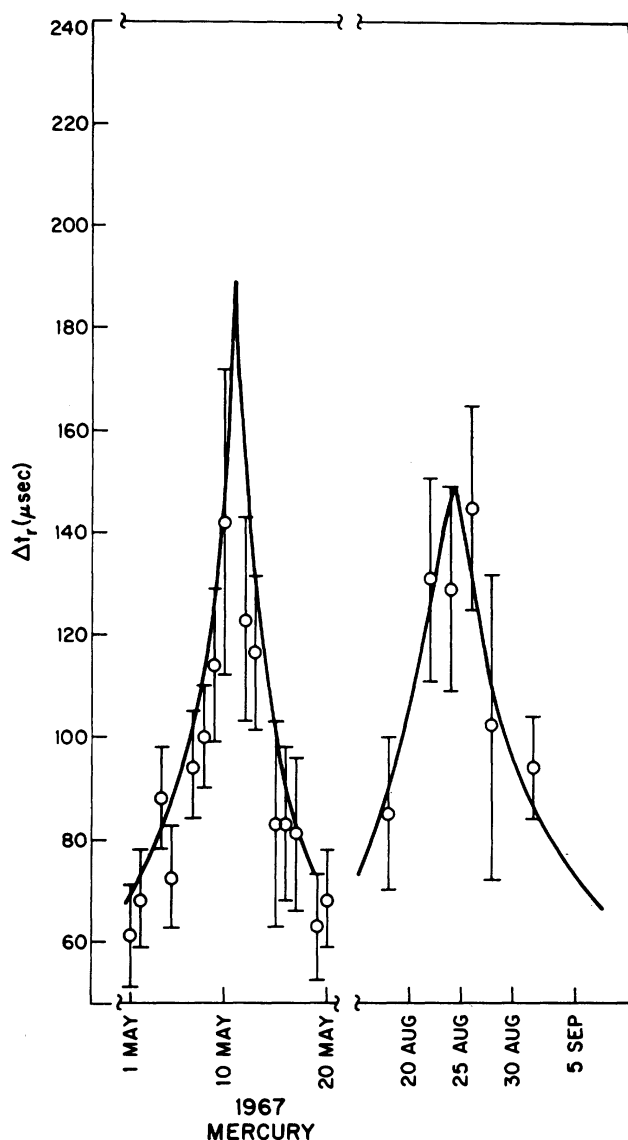


FIG. 16. Comparison of radar delay residuals for Mercury with respect to the Newtonian solution. Solid lines are predictions of the excess delay introduced by general relativity (fig. 11) (after Shapiro *et al.*, 1968).

TABLE 5  
ESTIMATES OF THE ROTATION PERIOD OF VENUS

Date of observation	Period (days)	Pole $\alpha$	Position $\delta$	Method (see footnote)	Author
1961	$\sim 225 ?$ +40	?	?	(1)	Victor <i>et al.</i> (1961)
1962	-230 -50	?	?	(1) (3)	Goldstein and Carpenter (1963)
1964	-249 $\pm$ 7	+10 75	68 $\pm$ 4	(1) (3)	Carpenter (1966)
1964	-244 $\pm$ 2	-4 90.9 $\pm$ 1	66.4 $\pm$ 1	(2)	Dyce <i>et al.</i> (1967)
1964	-241 $\pm$ 1	94 $\pm$ 3	-64 $\pm$ 2	(2)	Shapiro (1967a)
1966	-242.6 $\pm$ 0.6	?	?	(3)	Goldstein (1967)
1966	-243.09 $\pm$ 0.18	84.7 $\pm$ 1.8	65.8 $\pm$ 1.2	(2) (3)	Shapiro (1967b)

Methods (discussed in text): (1) base bandwidth, (2) delay-Doppler, (3) feature.

the signals from the limbs can be recognized in a plot of echo power *vs.* frequency, then  $\omega_T$  can be determined *via* Eq. (5). Unfortunately, these components of the echo are usually extremely hard to recognize in the presence of the noise and as a result there is a tendency for  $f_o$  to be underestimated.

A method which avoids this difficulty is to determine the frequency spectrum of the echoes reflected from a given annulus on the surface (fig. 6) defined by transmitting a pulse. In this case, the strongest portions of the power spectrum are the wings because of the fact that the Doppler contours become tangent to the annulus for the largest frequency shifts. This leads to a more reliable identification of the maximum frequency shift, and by restricting the radius of the annulus (defined by the delay of the pulse), echoes of adequate signal-to-noise ratio can be selected. The effects of the pulse width and other characteristics of the radar on the determination have been discussed by Shapiro (1967a).

Finally, there is the "feature method" in which the motion of an anomalously scattering region across the planetary disk is observed and the time of its reappearance noted. This method is akin to that employed optically and is intrinsically the most accurate since the accuracy increases directly in proportion to the number of rotations observed.

The observed angular rotation  $\omega_T$  is given by the vector sum of  $\omega_a$  and  $\omega_s$ , where  $\omega_s$  is the sidereal rotational angular velocity and  $\omega_a$  the apparent rotational angular velocity of the planet caused by the motion in space of the planet and of the radar. Although  $\omega_a$  is known, a single determi-

nation of the magnitude of  $\omega_T$  is not sufficient to define  $\omega_s$  since the pole position will not be known. Thus, there will be uncertainty in the magnitude and sense of rotation, unless the observations can be continued over a sufficiently long period such that the planet is viewed from many directions (Shapiro, 1967a).

#### B. OBSERVATIONS OF VENUS

The first radar observations of Venus (Victor *et al.*, 1961; Pettengill *et al.*, 1962) showed only that the Doppler spectrum of Venus was exceedingly narrow, pointing to a slow (possibly retrograde) rotation. During the second conjunction, Carpenter (1964) and Goldstein (1964b) found from the observations at the Jet Propulsion Laboratory that the rotation is retrograde, with a period of  $250 \pm 36$  days. The direction of motion could be inferred by the fact that the component  $\omega_a$  due to the motion of Venus past the earth is at a maximum near inferior conjunction and should serve to cause the Doppler broadening to be a maximum at this time if the rotation is direct. Kotelnikov *et al.* (1963b) also found a retrograde rotation, with a period in the range of 200 to 300 days, from observations in 1962. These results have been confirmed in subsequent observations at the Jodrell Bank Experimental Station, Millstone Hill radar, and the Arecibo Ionospheric radar. The uncertainty associated with the value of the rotation period has steadily decreased as indicated in table 5, largely as a result of the detection of features which reappear at succeeding inferior conjunctions. By combining this method with the delay-Doppler technique, Shapiro (1967b) has obtained a period of  $243.09 \pm 0.18$

days. The pole position is placed within  $2^\circ$  of perpendicular to the plane of the orbit of Venus, and nearly perpendicular also to the ecliptic plane.

These results are remarkable in two respects. With the exception of Uranus, no other planet in the solar system is known to execute retrograde rotation, and the period obtained implies capture by the earth. A rotation period of  $-243.16$  days would cause Venus to present exactly the same face toward earth at successive inferior conjunctions, with Venus executing four axial rotations as seen from earth between each encounter. It is extremely difficult to account for this earth-locked rotation without postulating a fractional difference in the equatorial moments [conventionally defined as  $(B - A)/C$  where  $A < B < C$  are the three principal moments] exceeding  $10^{-4}$  (Shapiro, 1967*b*; Goldreich and Peale, 1966*a, b*). In addition, it seems that in order to avoid capture by the sun (i.e., become sun-synchronous) the rotation must have initially been retrograde with a period somewhat shorter than its present value.

In sum, it seems that on Venus the sun rises in the West and sets in the East, the length of the solar day being 117 earth days. This occurs for reasons we do not fully understand, and the rotation seems to be locked at this period by a small couple exerted by earth at each inferior conjunction.

#### C. OBSERVATIONS OF MERCURY

Based upon optical observations of faint visual markings, Mercury was long thought to be in a sun-synchronous orbit, i.e., having an axial rotation period of 88 days (equal to its orbital period) so that the same face is always turned toward the sun. However, this view was discredited at Arecibo by Pettengill and Dyce (1965), Dyce *et al.* (1967), who determined that the rotation period is  $59 \pm 3$  days by radar using the delay-Doppler technique.

Following this result, the earlier optical data were re-examined by McGovern *et al.* (1965) who found that by and large these could be reconciled with a period of about 59 days. Colombo (1965) suggested that Mercury rotates with a period of 58.64 days, i.e., exactly two-thirds of the orbital period, and Colombo and Shapiro (1966) showed that the optical data were consistent with this. Thus, instead of executing one axial rotation per Mercury "year" the planet executes  $1\frac{1}{2}$  rotations as shown in figure 17. This means that at each perihelion passage an alternate

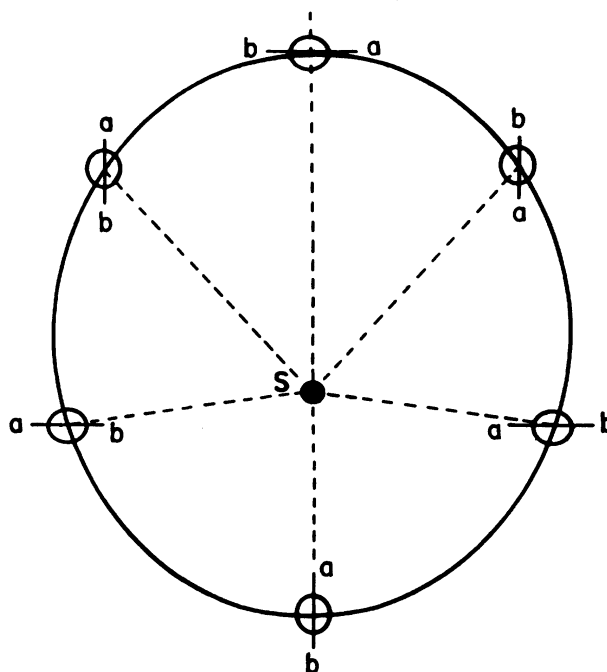


FIG. 17. Expected orientation of Mercury's axis of minimum moment of inertia as a function of orbital position. The spacing between the positions shown corresponds to equal intervals of time.

face is presented toward the sun. The stability of this motion appears to be brought about by the large eccentricity of Mercury's orbit. This causes the solar tidal torque to be much larger at perihelion than elsewhere (Peale and Gold, 1965), and acting in concert with the torque arising from any existing asymmetry leads to a stable spin state (Shapiro, 1967*c*).

#### VII. SUMMARY

In the brief period since the first detection of a planet by radar in 1961, much has been accomplished. The size of the solar system has been refined by five orders of magnitude and the elements of the orbits of Mercury, Venus, and earth have been improved considerably, based upon radar distance measurements. In addition, the radii and rotation rate of Venus and Mercury have been determined, with surprising results. Venus is found to rotate in a retrograde direction with a period of 243 days, its spin period apparently locked to cause the same face to turn toward earth at each inferior conjunction. Mercury on the other hand is locked to the sun, but executes  $1\frac{1}{2}$  rotations on its axis per orbital rotation instead of one axial rotation as was previously thought.

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