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Earth rotation measurement with micromechanical yaw-rate gyro

Rumen Arnaudov and Yasen Angelov

Faculty of Communications and Communications Technologies,
Technical University of Sofia, 8 Kliment Ohridski Street, Sofia, Bulgaria

E-mail: ra@tu-sofia.bg and yasen.angelov@dir.bg

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Abstract
The direct measurement of the Earth's rotation rate by means of a micromechanical yaw-rate gyroscope is difficult to achieve due to the considerable parameter variations of the current state-of-the-art sensors of this type. This paper outlines and applies a model of the external factors' effect on the sensor measurement via a method for their compensation through a mechanical change in the sensor's orientation. This allows the determination of a value such as the Earth's rotation rate, which is at the limit of the sensor's sensitivity and less than its short-term stability. A specialized information-measurement system has been developed for the implementation of the method. This system has been used for a number of measurements, presented in a graphical form. As a result, an average value of the Earth's rotation rate has been derived. This method is applicable for a subjective categorization and evaluation of micromechanical gyroscopes using a natural source of a very low angular speed.

Keywords: Earth's rotation measurement, drift compensation, micromechanical gyro

1. Introduction

The Earth's rotation is a fact, known to science for centuries. Many methods for the precise measurement of the Earth's rotation rate are used today, of varying complexity and levels of accuracy. Modern geodetic methods include VLBI (very long baseline interferometry), satellite systems such as GPS and GLONASS, laser ranging to artificial Earth satellites of the Lageos or Starlette (satellite laser ranging), LLR (lunar laser ranging), etc [1].

Micromechanical inertial sensors—accelerometers and gyroscopes—have undergone significant development and a steady improvement in performance during the last few decades. The Earth's rotation rate is $\psi_E = 4.178074 \times 10^{-3}$ deg s^{-1} according to WGS84 (World Geodetic System 1984). This value is of the order of the threshold sensitivity and accuracy of contemporary micromechanical (Coriolis) gyroscopes [2–4]. Therefore, we are looking for ways of using a micromechanical gyroscope for measurements of the Earth's rotation rate.

However, the direct use of such inertial sensors in the determination of ψ_E is practically impossible. The main reason for this is the fluctuation of output in temporal terms, their complex dependence on temperature changes and sensitivity to linear accelerations. These deficiencies prompt the need to develop special methods and algorithms, or else the influence of these factors will prevent the accurate measurement of ψ_E. The current paper elaborates such a special method and algorithm.

2. The measurment of the Earth's rotation rate with a micromechanical gyroscope

Micromechanical gyros (or MEMS gyros) are usually designed as an electronically driven resonator, often made out of a single piece of quartz or silicon. Such gyros operate in accordance with the dynamic theory that when an angular rate is applied to a translating body, a Coriolis force is generated [3]. This force, which is proportional to the applied angular rate, causes displacement that can be measured capacitively in a silicon instrument or piezoelectrically in a quartz instrument. A lot of manufacturers are developing and offering gyroscopes and accelerometers based on this technology [5–7].

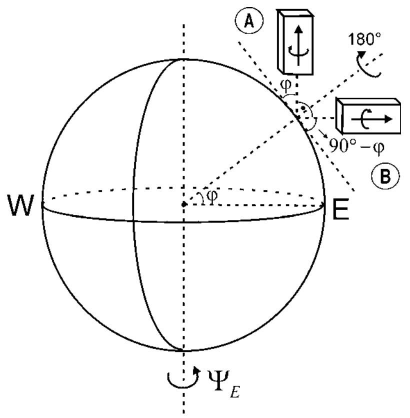

Figure 1. The general model of the Earth's rotation rate a measurement with a micromechanical gyroscope.

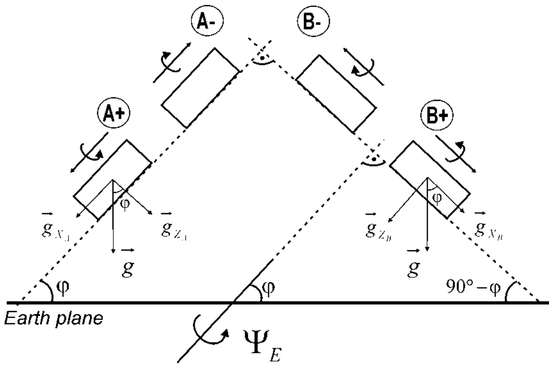

Figure 2. The model for the measurement of $\psi_{\rm E}$ with the change in sign of the measured quantity via re-orientating a single sensor

Generally, for the measurement of the Earth's rotation rate $\psi_{\rm E}$ (figure 1) the sensitive axis of the gyroscope (marked with an arrow in figure 1) must be situated in parallel with the Earth's axis (position A). On the other hand, to determine the sensor's null value, it is necessary for the sensor to take a position in which there is no input signal (rotation rate)—the sensitive axis of the sensor is perpendicular to the Earth's axis (position B).

If the measurement is carried out on the equator, taking up these two positions consists in orienting the sensor's sensitive axis parallel to the Earth's surface while pointing to the north. As the real measurements are carried out at a latitude of φ°N (figure 1), the sensor (pointing northwards) must be situated along a plane which is slanted at an angle of φ to the Earth's surface (position A). When the sensor is rotated 90° (position B) there is a zero input signal. Since such a rotation is difficult for practical realization, it is better to rotate the whole experimental setting 180° along the plane of the Earth's surface (figure 2) and change the angle of the slant to $90^\circ - \varphi$.

The measurement of $\psi_{\rm E}$ according to the scheme proposed in figure 1 is easy to utilize with an ideal sensor. In reality, however, phenomena [8] such as changes in bias, temperature dependence, sensitivity to linear acceleration, noise etc are observed in real sensors.

A special focus of attention must be linear acceleration sensitivity, because the influence of gravity varies according to the position of the sensor. The presence of noise requires many measurements to be carried out over a long period of time and statistically processed to achieve a more precise estimate. On the other hand, long-term repeated experiments strengthen the influence of the other factors in time.

To decrease the influence of these factors, a method [9] is used and developed whereby the drift of the sensor is compensated for by a change in the sign of the measured parameter. This change is achieved through a mechanical rotation of the sensor. The scheme from figure 1 is further developed so that for each position A and B the single sensor is mechanically rotated, thus changing the sign of the measured quantity. This process is illustrated in figure 2.

In describing the output signal of the sensor, we use the suggested overall (summary) model [10], to which we have added the sensitivity to linear accelerations:

$$\psi = \overline{\psi} + \Delta\psi_{\rm DC} + \Delta\psi_{\rm LA} + \nu, \tag{1}$$

where ψ is the value measured by the sensor, $\overline{\psi}$ is the real-life rotation rate that should be measured, $\Delta\psi_{\rm DC}$ is the zero offset, $\Delta\psi_{\rm LA}$ is the total influence of the linear accelerations upon all axes and ν is the Gaussian noise. Taking into account the components of the Earth's acceleration $\vec{g}$ on the sensor's axes and equation (1), we have evolved the following system of equations for positions A+, A−, B+, B−:

$$\left|\begin{array}{l} \psi_{\rm A+} = \Delta\psi_{\rm DC} + \Delta\psi_{\overrightarrow{g\rm ZA}} + \Delta\psi_{\overrightarrow{g\rm XA}} + \psi_{\rm E} \\ \psi_{\rm A-} = \Delta\psi_{\rm DC} + \Delta\psi_{\overrightarrow{g\rm ZA}} - \Delta\psi_{\overrightarrow{g\rm XA}} - \psi_{\rm E} \\ \psi_{\rm B+} = \Delta\psi_{\rm DC} + \Delta\psi_{\overrightarrow{g\rm ZB}} + \Delta\psi_{\overrightarrow{g\rm XB}} + 0 \\ \psi_{\rm B-} = \Delta\psi_{\rm DC} + \Delta\psi_{\overrightarrow{g\rm ZB}} - \Delta\psi_{\overrightarrow{g\rm XB}} + 0, \end{array}\right. \tag{2}$$

where $\Delta\psi_g$ denote the influence of the components of $\vec{g}$ upon the conditionally defined axes x and z.

The zero offset $\Delta\psi_{\rm DC}$ in equation (2) is assumed to be changing insignificantly among the measurements in the different positions, because of their relatively rapid change (every 2–3 s). Hence $\Delta\psi_{\rm DC}$ is denoted by the same variable in all equations. As a result of the subtraction of equations from (2) (the second is subtracted from the first and the fourth from the third) we get

$$\left|\begin{array}{l} \psi_{\rm A+} - \psi_{\rm A-} = 2\,\Delta\psi_{g_{\rm XA}} + 2\,\psi_{\rm E} \\ \psi_{\rm B+} - \psi_{\rm B-} = 2\,\Delta\psi_{g_{\rm XB}}. \end{array}\right. \tag{3}$$

Therefore

$$\psi_{\rm E} = \tfrac{1}{2}\left[(\psi_{\rm A+} - \psi_{\rm A-}) - (\psi_{\rm B+} - \psi_{\rm B-})\right] - G + \nu, \tag{4}$$

where

$$G = \Delta\psi_{g_{\rm XA}} - \Delta\psi_{g_{\rm XB}}.$$

In (4) the main factor of negative influence on the measurement—the bias drift—is excluded, as well as its temperature dependence. This gives an opportunity for long-term measurements to be carried out without applying additional measures for temperature compensation or stabilization. Thus a large number of data can be averaged by minimizing the effect of the noise on the measurement and achieving a better approximation of $\psi_{\rm E}$.

The expression G reflects the sensitivity of the sensor to linear acceleration along the x axis. Its influence is best evaluated when transformed as follows:

$$G = k_x g \sin\varphi - k_x g \cos\varphi = k_x g(\sin\varphi - \cos\varphi), \tag{5}$$

where k_X (deg s^{-1} g^{-1}) denotes the coefficient of the influence of the linear acceleration along the x axis.

Table 1. The main specifications of the HZ1-90-100A sensor.

Parameter	Value
Standard range	± 90 deg s^{-1}
Scale factor ($\pm 2\%$)	22.2 mV deg s^{-1}
Resolution and threshold	<0.004 deg s^{-1}
g sensitivity (x axes)	<0.006 deg s^{-1} g^{-1}
Bias short-term stability (100 s, constant temperature)	<0.05 deg s^{-1}

It is obvious that a complete neutralization of this sensitivity is achieved only when $\varphi = 45°$ ($G = 0$). Upon moving towards the equator or the poles ($\varphi = 0$, $\varphi = 90°$) G increases and reaches the values of $\pm k_X g$. In order to calculate G it is necessary to know the value of k_X for every individual sensor and latitude.

The measurements of groups A and B may be separated in time, because the zero bias, which is important for each group, is in fact eliminated in the last equations (3). Perhaps it will be more precise to denote the zero offsets with different variables, $\Delta\psi_{\mathrm{DC_A}}$ and $\Delta\psi_{\mathrm{DC_B}}$, but eventually it will not change the equation system (3). Still, long separation in time should be avoided, because it can boost the influence of the other negative factors and reduce the correlation in the sensor's condition.

3. Practical results

The proposed method is implemented by a micromechanical gyroscope of type HZ1-90-100A of Systron Donner [5]. This sensor is chosen because its resolution value and sensitivity threshold (table 1) are of the same order as the measured value ψ_{E}. Nevertheless, the direct measurement of ψ_{E} is impossible because of the limited short-term stability of the sensor.

The latitude of the actual measurement location is $\varphi \approx 42.6°$N (Sofia, Bulgaria). Taking into account the worst case, when the maximum value of linear acceleration sensitivity $k_X = 0.006$ deg s^{-1} g^{-1}, G is calculated as follows:

$$G_{\varphi=42.6°} = -0.0592\, k_X\, g < -3.553 \times 10^{-4}\ \mathrm{deg\, s^{-1}}. \quad (6)$$

The calculated value for G is one order less than the Earth's rotation rate and initially can be ignored so that the measurement is reduced to

$$\psi_{\mathrm{E}} = \tfrac{1}{2}\left[(\psi_{\mathrm{A+}} - \psi_{\mathrm{A-}}) - (\psi_{\mathrm{B+}} - \psi_{\mathrm{B-}})\right]. \quad (7)$$

The utilization of this method has been achieved via the information-measurement system shown in figure 3. It performs all the routines for the stepper motor control (for consecutive altering positions '+' and '−') interface to the analogue to digital converter and to the external personal computer. Only the rotation of the sensor at positions '+' and '−' is carried out automatically by means of the stepper motor, while A and B alternate positioning is performed by hand. With the aim of simplifying the experiment a lot of measurements are carried out first, at position A, and then at position B. At each individual position ('+' or '−') 256 samples at 2 ms rate are made and then averaged. The total time for a full '+'/'−' cycle amounts to about 1.79 s, which includes the time needed for the stepper motor to rotate.

Under the described conditions some long-term measurements have been carried out at room temperature on different days within a week. Each measurement consists of accumulating data from M cycles ($M = 3000$–$10\,000$) at each position A+− and B+−. In figures 5–10 two of the measurements are represented graphically.

The calculated average value for all measurements of the Earth's rotation rate according to equation (7) amounts to

$$\psi_{\mathrm{E}} = 0.004\,54\ \mathrm{deg\, s^{-1}}.$$

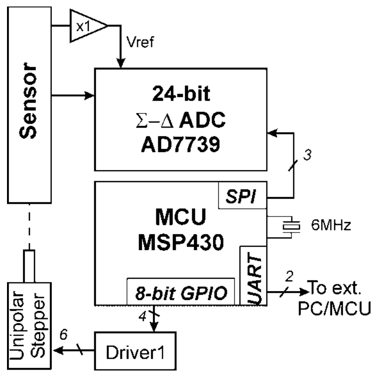

Figure 3. Block diagram of the information-measurement system for the measurement of the Earth's rotation rate.

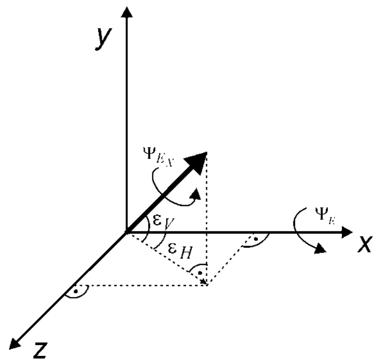

Figure 4. A simplified model of the influence of the positioning error on the applied method.

4. Analysis of experimental data and sources of error

These sensors exhibit a typical drift in the output, well observed in the measurements illustrated in figures 5, 6, 8 and 9. The drift shows values of 0.05–0.3 deg s^{-1}, which obviously makes it impossible to perform a direct measurement of a value such as the Earth's rotation rate. Applying the proposed method and equation (4) (figures 7 and 10) the drift is compensated for and the calculated value of ψ_{E} approaches the specified WGS84 value. The increased number of measurements results in a more accurate estimate of the Earth's rotation rate because it decreases the influence of noise.

In spite of the effectiveness of the proposed method, the measured value does not exhibit a very high accuracy, $\eta \approx 8.7\%$, and this is due to some basic factors.

The most important error source is the scale factor of the gyroscope. Although for the sake of clarity the scale factor is

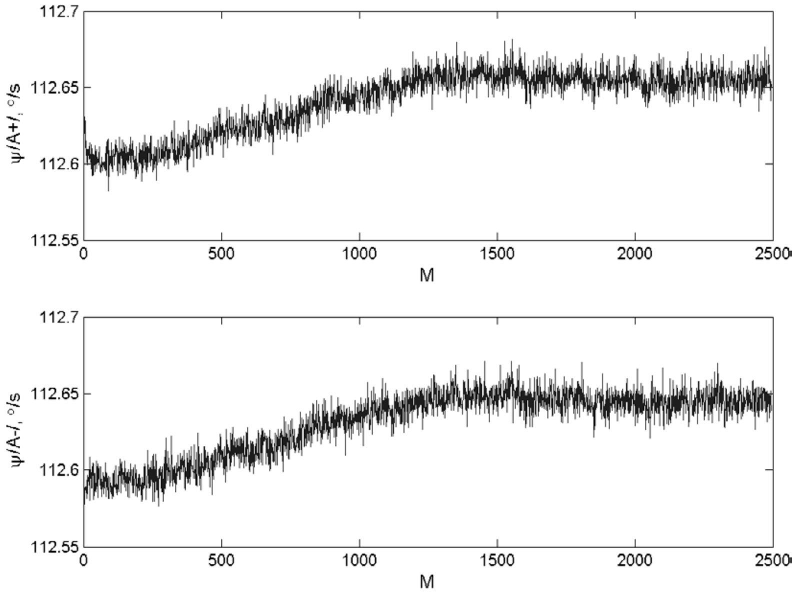

Figure 5. The measured rotation rate from the sensor at positions A+ and A−.

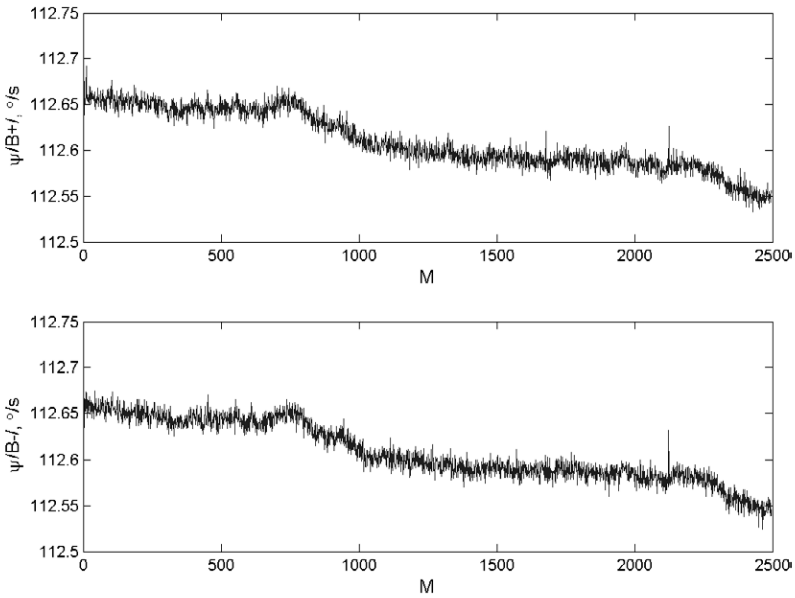

Figure 6. The measured rotation rate from the sensor at positions B+ and B−.

not presented in the above equations, it participates in them all. The scale factor is factory calibrated with a precision of 2% and additionally has a temperature dependence <0.08% $^{\circ}C^{-1}$. Its influence on the measurements is not compensated for by this method and it directly impacts the final result. Since the experiments have been carried out at room temperature, which does not vary much, the temperature dependence is not a decisive factor (for this particular sensor a 5 °C deviation leads to just a <0.4% error in the scale factor value).

The initial positioning to the north at slants of φ and $\varphi - 90^{\circ}$ is performed without any special precise measuring instruments, which also has affected the final error of the measurement. Here is a brief analysis of the influence of errors in orientation on the final result.

The positioning errors are divided into three types:

(1) ensuing from the stepper motor rotation at 180°;
(2) ensuing from the positioning northwards;
(3) ensuing from situating the plane at an angle of φ or $\varphi - 90^{\circ}$.

The angular position accuracy of a stepper motor varies from one step to the next. Manufacturers can achieve ±5% fundamental step accuracy [11]; for instance, such an error with a 7.5° per step motor leads to an angle error of $\varepsilon_S = 0.375^{\circ}$. If we study this error in isolation, it triggers off a $\cos(\varepsilon_S)$ shift in the vector component of the Earth's rotation along the sensitive axis of the sensor, which in this case is equivalent to a mere 0.002% error.

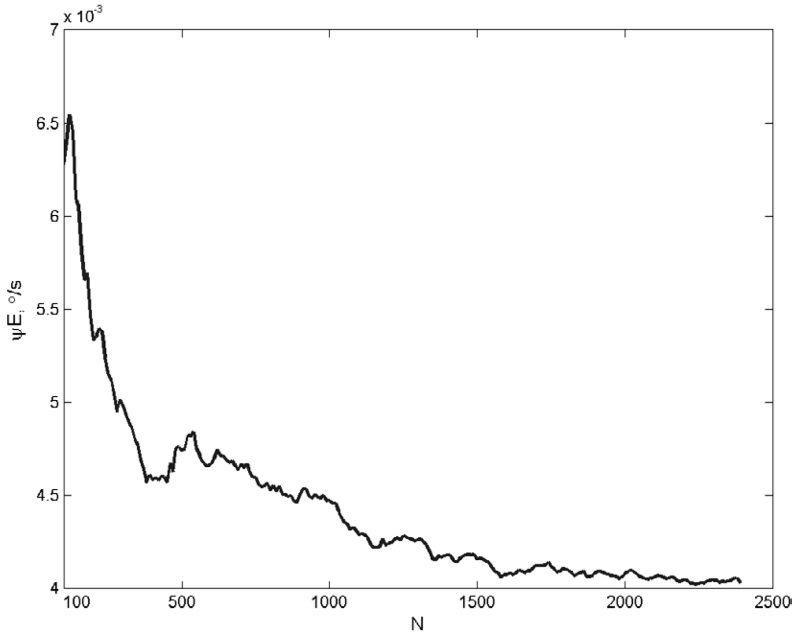

Figure 7. The average value of ψ_E as a function of the number (N) of measurements.

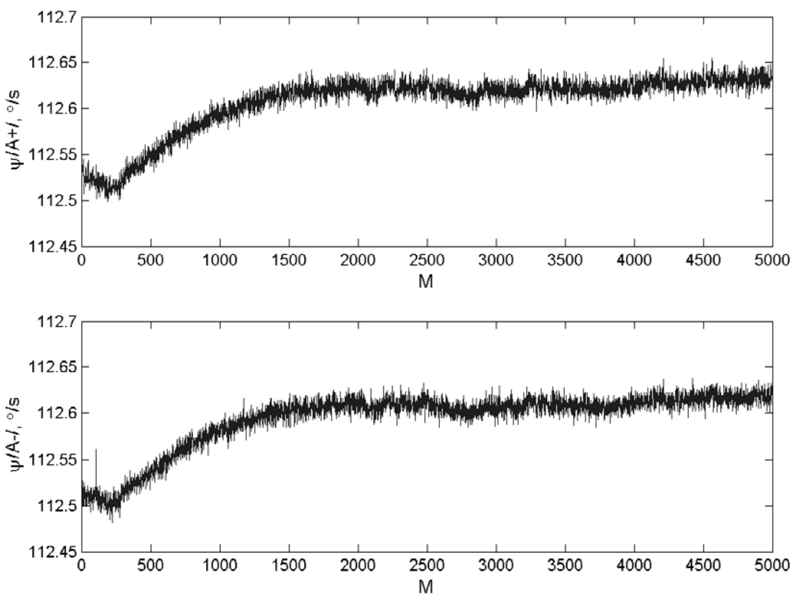

Figure 8. The measured rotation rate from the sensor at positions A+ and A−.

Figure 4 illustrates a simplified model of the influence of the last two errors. The positioning errors (ε_V and ε_V) are defined in relation to an ideally oriented coordinate system—the x axis coincides with the ψ_E vector for positions A+ and A−, and the plane $\{x, z\}$ coincides with the plane described in section 2. Every deviation from the ideal position leads to changes in the vector component of the Earth's rotation along the active axis according to the equation

$$\psi_{E_A} = \cos\varepsilon_H \cos\varepsilon_V \,\psi_E. \tag{8}$$

Any deviation from the ideal position at B+ and B− incurs a similar situation; however, the Earth's rotation rate vector becomes perpendicular to the sensitive axis, so the vector component is determined in the following way:

$$\psi_{E_A} = \sin\varepsilon_H \sin\varepsilon_V \,\psi_E. \tag{9}$$

Substituting equations (8) and (9) in equation (2) and applying the above-mentioned sequence of transformations we derive the following final equation:

$$\psi_E = \frac{1}{2(\cos\varepsilon_H \cos\varepsilon_V - \sin\varepsilon_H \sin\varepsilon_V)} \times [(\psi_{A+} - \psi_{A-}) - (\psi_{B+} - \psi_{B-})]. \tag{10}$$

At this point we should underscore that equation (10) shows only a simplified and approximate analysis of these errors. In general, these errors lead to a shift in the spatial orientation

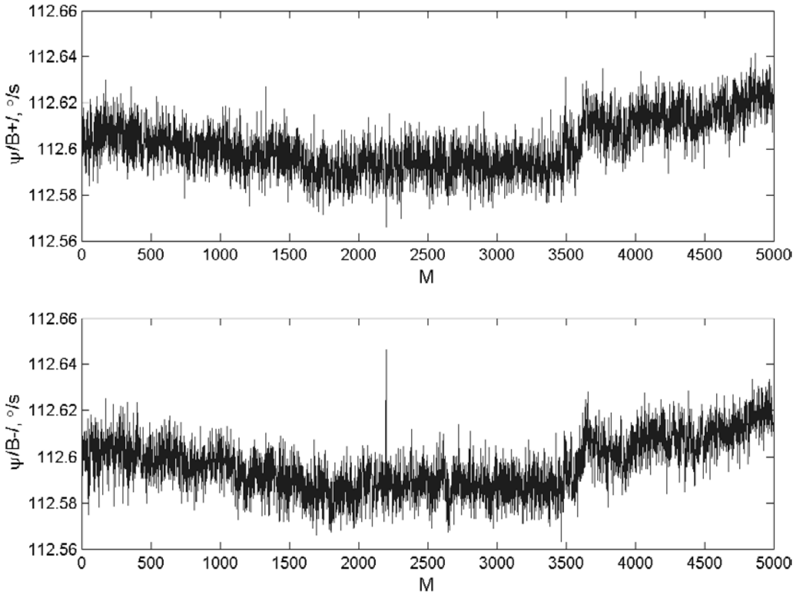

Figure 9. The measured rotation rate from the sensor at positions B+ and B−.

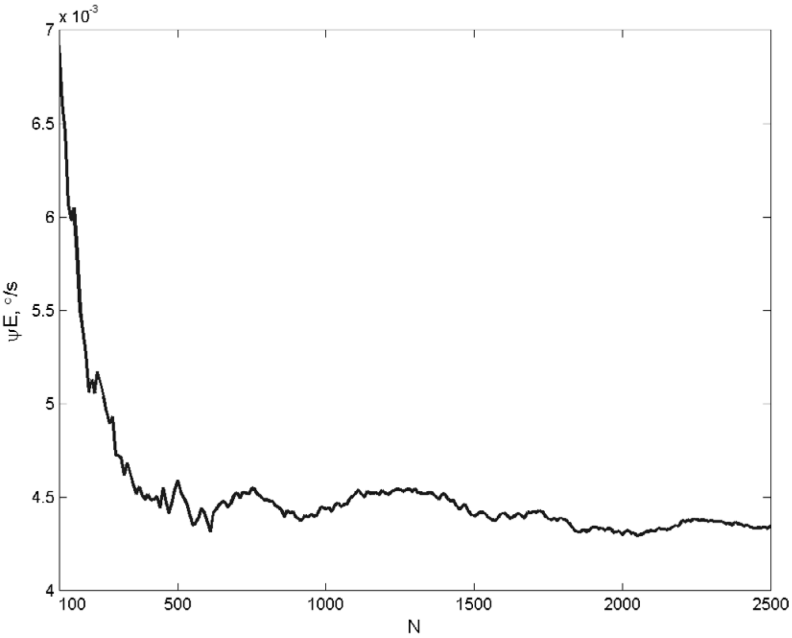

Figure 10. The average value of ψ_{E} as a function of the number (N) of measurements.

of the sensitive axis. Therefore, a detailed solution to the problem requires more complex mathematical operations, based on Euler's transformation in 3D [12]. This problem goes beyond the scope of the current research. What is much more important, though, is the fact that $\varepsilon_{\mathrm{V}} = \varepsilon_{\mathrm{H}} = 1^{\circ}$ leads to an error of less than 0.1% in the calculated value as per equation (7).

Apparently, all inaccuracies in the sensor's orientation cause, on the one hand, different impacts on the other axes of the sensor and, on the other hand, some changes in the influence of gravity (the equation for G). But sensitivity along the other axes and the value of G are considerably smaller than the measured value of the Earth's rotation rate; therefore, they can be ignored.

5. Conclusion

This paper presents a novel method for the measurement of the Earth's rotation rate with a micromechanical gyroscope. The main advantage of this method is the compensation of the instability of the sensor's output value which, in turn, allows the measurement of values much smaller than the instability value itself. The scientific contribution consists of the analysis offered and the developed model of the influence factors. What is more, a final equation has been derived with the aim of determining the Earth's rotation rate from the experimental data. A brief analysis of the impact of positioning errors on the measurements is carried out. The calculated value of the Earth's rotation rate is 0.004 54 deg s^{-1}, which amounts to a total error of 8.7%.

The proposed method can be used mainly for a subjective categorization and evaluation of different brands of micromechanical gyroscopes.

The proposed method does not require the construction of a complicated experimental setting to achieve a very low and stable angular speed. 'And still, the Earth rotates' [Galileo Galilei]!

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